Iranian Journal of Geophysics, Vol 15 No 4, 2022, Page 29 – 36 (Research Article)
DOI: 10.30499/IJG.2021.260423.1306

Calibration of the accelerometers on board GRACE satellites using discrete wavelet transform

Amirreza Moradi [1*] and Mohammad Ali Sharifi [2]

[1] *Assistant Professor, Department of Surveying Engineering, Arak University of Technology, Arak, Iran*
[2] *Associate Professor, School of Surveying and Geospatial Engineering, College of Engineering, University of Tehran, Tehran, Iran*

(Received: 06 December 2020, Accepted: 02 Febreury 2021)

Abstract

The Gravity Recovery and Climate Experiment (GRACE) mission and its successor, GRACE Follow-On, have been observing the Earth's static and time-variable gravity field with unprecedented accuracy from 2002, thanks to the precision equipment used, such as very accurate ranging systems, dual-frequency GPS receivers, star cameras and highly sensitive accelerometers. The accelerometers on board of these missions provide high quality measurements of the non-gravitational forces acting on the satellites, provided that they are calibrated. In this paper, a wavelet-based detrending scheme is used to estimate drift and bias of GRACE accelerometer data. This method is applied to a simulated noisy time series and two sets of GRACE accelerometer data (recorded on January 1, 2005 and during March 2015). The results confirm the speed and ease of the proposed method due to the nature of the wavelet-based detrending scheme. The estimated bias and drift parameters have acceptable accuracies because the wavelet-based method does not require any reference value and its results are not affected by uncertainties in gravitational field modeling. Furthermore, some computational problems such as the amplification of noise during the numerical differentiation of satellite positions do not exist. In addition, the accelerometer readouts, provided in the Science Reference Frame (SRF), may be calibrated without applying any coordinate transformation.

Keywords: GRACE, accelerometer calibration, wavelet transformation

* Corresponding author: amoradi90@arakut.ac.ir

1 Introduction

The Gravity Recovery and Climate Experiment (GRACE) mission was launched in 2002 (Tapley et al., 2004). From then on, it mapped the Earth's static and time-variable gravity field with unprecedented accuracy. This mission consisted of two co-orbiting spacecrafts with a separation of about 220 km at an orbital attitude of about 500 km. Each spacecraft carried a dual one-way microwave ranging system with a precision of ± 0.1 µm/s in range-rate, dual-frequency GPS receiver, star cameras, accelerometer, and SLR reflector, providing the observations mainly constituted the products labeled L1B. These products consisted of the quantities such as the intersatellite range, range rate, range acceleration, the non-gravitational acceleration on each satellite, the orbits and so on (Chen, 2007). All conventional products of the GRACE, i.e., the monthly gravity field estimates in the form of spherical harmonic coefficients and Mascon solutions (Lemoine et al., 2007), as well as other gravity products recovered globally or regionally (Chen, 2007), have been obtained on the basis of L1B data processing, with especial attention to the nongravitational accelerations acting on each spacecraft. For this purpose, the Level-1B accelerometer (ACC1B) data had to be calibrated because they were influenced by the instrument bias, drift and scale. Weigelt (2007) showed that the disturbing potential series along GRACE orbits could be deviated from a constant level with about 1100 m^2/s^2 per day because of using the un-calibrated accelerometer data. Moreover, satellite accelerometer data could contain temporal variations including a linear trend coming from fluctuations in the thermospheric neutral densities. Such variations are of special interest during the geomagnetic storms (Vielberg et al., 2018). Therefore, it is also important to determine the linear trend in satellite accelerometer time series in terms of their physical contents. These methods may be generally classified into two classes, acceleration approaches and energy balance approaches, corresponding to the general methods of recovering the GRACE gravity products. In the first category, the total satellite accelerations derived from Precise Orbit Determination (POD) are used to estimate the non-gravitational accelerations by subtracting the modeled gravitational accelerations from the total accelerations as a standard to compute the GRACE accelerometer calibration parameters in the least-squares sense (for further details see for example, Van Helleputte et al., 2009; Bezdek, 2010; Chen, 2007; Švehla and Földváry, 2006).

In the energy balance approach, the relation between the true non-gravitational accelerations and the accelerometer measurements are integrated along the satellites orbit to produce the dispersive energy equation as the observation equation for least-squares estimation of the calibration parameters (see for example, Chen, 2007; Weigelt, 2007; Tangdamrongsub et al., 2012).

Both acceleration and energy integral methods need to reference values of non-gravitational field affected by uncertainties in gravitational field modeling and also by some computational problems such as the amplification of noise during the numerical differentiation of satellite positions. To these must be added the difficulties of some essential transformations between the inertial and the spacecraft frames.

According to Vielberg et al. (2018), which compared different GRACE accelerometer calibration procedures to evaluate the impact of these methods on the estimation of global thermospheric neutral densities, the scale factor of the uncalibrated accelerometer data can almost be considered constant for all methods. Correspondingly, in case of using direct GRACE L1B measurements such as range rates to analyze the behavior of the related time series, instead of converting them to conventional gravity products, the scale factor is negligible. For instance, Han et al. (2009) analyzed the contribution of different terrestrial water storage components such as soil moisture and surface water in GRACE inter-satellite range rate data over the Amazon area, and Moradi and Sharifi (2016) used these observations to extract the time-frequency behavior of irregularly sampled GRACE range rate time series related to Iran's main catchments. In this study, we propose a method of calibrating the accelerometer measurements with emphasis on determining the bias and drift parameters, based on the approach of using wavelets to identify trends in time series introduced by Andreas and Treviño (1997). They showed that the discrete wavelet transform of a time series containing a quadratic trend when using the inverted Haar wavelet or the Elephant wavelet as the base functions, could provide the coefficients of the trend polynomial more than twice as fast as the least-squares estimation procedure.

Here, this method is applied for estimating the drift and the bias of the accelerometer measurements by considering them as the trend parameters in the non-gravitational acceleration time series. The method is capable of direct estimation of the drift and bias parameters as fast as possible with no reference values for the non-gravitational accelerations. The next sections of this paper are organized as follows. At first, in addition to a brief overview on detecting trends by wavelets, we describe how this scheme is applied for estimating the drift and the bias of the accelerometer measurements. Next, the used data are described and the results are presented and compared.

2 Mathematical foundations

2-1 A brief overview on using wavelets to detect trends

A trend in a time series can be defined as its any component with a period longer than the length of the signal (Andreas and Treviño, 1997). In other words, the trend is the component that represents the low frequency variations in a time series. Generally, the relation between the instantaneous value of a time series $\tilde{g}(t)$ measured only for $0 \leq t \leq L$, and its zero mean (trendless) component, $g(t)$, is modeled as:

$$\tilde{g}(t) = g(t) + \mu_0 + \mu_1 t \qquad (1)$$

where μ_0 and μ_1 are the coefficients of the trend polynomial, which have to be estimated and then isolated. To do this, the wavelet based detrending procedure, proposed by Andreas and Treviño (1997), starts by convolving the measured time series $\tilde{g}(t)$ with the inverted Haar wavelets as the base functions to generate the discrete wavelet coefficients of the time series as follows:

$$WT(t,L) = \int_{-\infty}^{+\infty} \tilde{g}(s) I(s-t,L) ds \qquad (2)$$

where $I(t,L)$ is the inverted Haar wavelets (Fig. 1), which is expressed as:

$$I(t,L) = \begin{cases} -(\frac{2}{L})^2, & -L/2 \leq t \leq 0 \\ (\frac{2}{L})^2, & -L/2 \leq t \leq 0 \\ 0, & elsewhere \end{cases} \qquad (3)$$

Using Eq. (2) and computing the wavelet coefficient at $t=L/2$ gives:

$$WT(L/2,L) = (\tfrac{2}{L})^2 \int_0^{L/2} \left\{ \tilde{g}\left(s+\tfrac{L}{2}\right) - \tilde{g}(s) \right\} ds \qquad (4)$$

Substituting Eq. (1) for $\tilde{g}(t)$ in Eq. (3) and doing integration leads to:

$$WT(L/2,L) = (\tfrac{2}{L})^2 \int_0^{L/2} \left\{ g\left(s+\tfrac{L}{2}\right) - g(s) \right\} ds + \mu_1 \qquad (5)$$

Considering that the integral in Eq. (5) is almost zero, yields:

$$\hat{\mu}_1 = WT(L/2,L) \qquad (6)$$

On the other hand, introducing Eq. (1) into the sample average of the time series $\tilde{g}(t)$ gives:

$$\bar{\tilde{g}} = \frac{1}{L}\int_0^L \tilde{g}(s) ds \Rightarrow \bar{\tilde{g}} = \frac{1}{L}\int_0^L g(s) ds + \mu_0 + \frac{1}{2}\mu_1 L \qquad (7)$$

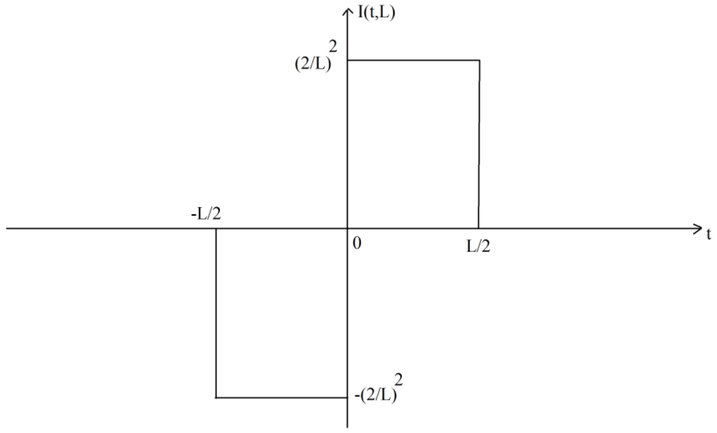

Fig1. Inverted Haar wavelet.

Again, since $g(t)$ should expectedly have zero mean, μ_0 is estimated as:

$$\hat{\mu}_0 = \bar{\bar{g}} - \frac{1}{2}\hat{\mu}_1 L \quad (8)$$

The focus of this study is just on the linear trend component of the time series due to the expected behavior of the GRACE accelerometer data, but the Eq. (1) and the wavelet-based detrending process can be extended to identify the probable quadratic trend according to Andreas and Treviño (1997).

2-2 Application to GRACE Level-1B accelerometer data

Based on the method of Kim (2000), the relation between the true non-gravitational accelerations (a_i^{true}) and the uncalibrated GRACE accelerometer measurements (a_i^{raw}) with respect to the satellite reference frame can be expressed by a bias (b_i), a drift (d_i) and a scale factor (s_i), for i=1, 2 and 3, corresponding to the components in the satellite reference frame (the along-track, the cross-track and the radial direction) as:

$$a_i^{raw} = s_i * a_i^{true} + b_i + d_i * t \quad (9)$$

Eq. (9) differs from Eq. (1) in that it contains the scale factor, so regardless of the scaled component, the bias and drift parameters can be estimated based on the proposed detrending scheme. To that end, Eqs. (2), (6) and (8) are rewritten as:

$$\begin{cases} WT(t,L) = \int_{-\infty}^{+\infty} a_i^{raw}(s) I(s-t,L)ds \\ \hat{d_i} = WT\left(\frac{L}{2}, L\right) \\ \hat{b_i} = \overline{a_i^{raw}} - \frac{1}{2}\hat{d_i}L \end{cases} \quad (10)$$

The estimation of the bias and drift parameters leads to a scaled non-gravitational acceleration signal (a_i^{scaled}) as a new data related to the true non-gravitational accelerations as follows:

$$a_i^{scaled} = s_i * a_i^{true} \quad (11)$$

In this study, the above scaling parameter may be assumed to be known; therefore, the proposed process is limited to estimating the bias and drift parameters to achieve the objectives mentioned earlier.

3 Data analysis and results

To test the proposed method, we firstly construct an artificial time series with the known bias and drift as:

$$\tilde{g}(t) = \sin(2\pi t) + 0.01t - 15.5 \quad (12)$$

with sampling rate of 50 samples/s and a duration of 10 seconds. The desired 1 Hz sine wave is supposed to be extracted by calibrating scheme. This part of $\tilde{g}(t)$ is the derivative of a cosine signal $f(t)$, which is used as a function to create the reference values for extracting the drift and bias parameters using the least-squares regression as an alternative to our proposed method. Since in practice, the reference values are affected by noise amplification resulted from numerical differentiation of the preliminary noisy data, the signal $f(t)$ is chosen as:

$$f(t) = -\frac{1}{2\pi}\cos(2\pi t) + \varepsilon \qquad (13)$$

where ε is an independent, normally distributed Gaussian noise.

On the one hand, we apply our proposed wavelet-based method to $\tilde{g}(t)$ in order to estimate the drift and bias parameters, and on the other hand, these parameters were identified by least-squares regression after comparing the test data $\tilde{g}(t)$ with the reference values obtained by differentiating $f(t)$. The estimated calibration values obtained by both wavelet-based and least-squares regression methods as well as their corresponding Root Mean Square Errors (RMSE) are shown in Table 1. Due to the effect of the quality of reference values on the results of the least-squares method, the additive noise amplitude in the reference signal $f(t)$ has been changed repeatedly to meet the lowest possible RMSE for the least-squares estimates of the bias and drift parameters. The best results have been shown in the table.

Table 1. Estimated calibration values for the test data $\tilde{g}(t)$ obtained by wavelet-based and least-squares regression methods as well as their corresponding Root Mean Square Errors (RMSE).

Calibration method	Estimated bias	Estimated drift	RMSE
Proposed wavelet-based method	−15.4999998007976	0.009999960159522	0.000000203147385
Least-squares fitting	−15.4074685418423	0.008540508343793	0.092542967667374

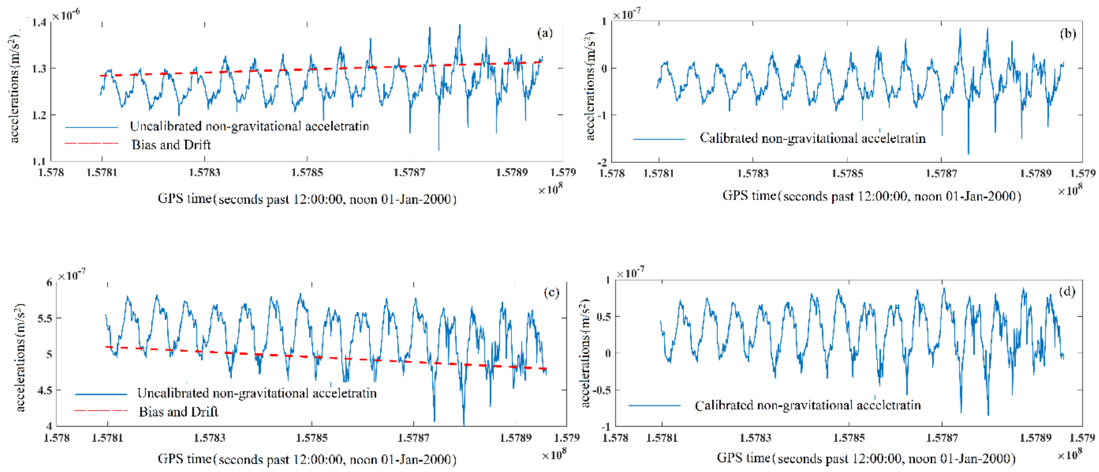

Fig 2. Acceleration measurements of GRACE A/B in the flight direction during January 1, 2005. (a) and (c) Uncalibrated non-gravitational accelerations and their corresponding wavelet-based estimated bias and drifts for GRACE A and B, respectively. (b) and (d) Wavelet-based calibrated non-gravitational accelerations for GRACE A and B, respectively.

Although according to Andreas and Treviño (1997) the least squares estimation is a little more accurate than the wavelet-based detrending scheme, but here, the damaging effect of noise amplification resulted from the numerical differentiation during the least-squares process makes the wavelet-based scheme more efficient from both computational speed (Andreas and Treviño, 1997) and accuracy points of view.

The second time series analyzed in this study is constructed using the non-gravitational accelerometer measurements (ACC1B) with one second sampling, included in the products labeled

L1B (GRACE LEVEL 1B JPL RELEASE 2.0. Ver. 2. PO.DAAC, CA, USA) (Case et al., 2002), which is available since mid-2002. Firstly, the non-gravitational accelerations recorded on January 1, 2005 are considered as the uncalibrated signal. Applying the wavelet-based method to daily time series for each satellite in each direction leads to estimate the corresponding bias and drift parameters. The raw signals, as well as the calibration parameters and the calibrated time series are shown in Fig. 2 and the estimated bias values that have been compared to the corresponding estimates of them according to Bettadpur (2009) are presented in Table 2. Since Bettadpur (2009) has just provided the bias parameter, the wavelet-based estimates of drift parameters have not been listed in Table 2.

Bettadpur (2009) has provided a long average of the bias parameter as the result of GRACE data analysis from the beginning of the mission until March 31, 2009, in two separate data spans (before and after March 7, 2003); thus, we have only expressed the above example in terms of validity and have examined the quality of the results in the next example more closely. For this purpose, the daily biases of acceleration measurements of GRACE-A in along-track, radial and cross-track directions of the Satellite Reference Frame (SRF) during March 2015, have been estimated using our proposed wavelet-based method and their corresponding mean values and standard deviations have been compared to those of provided by Vielberg et al. (2018), as shown in Table 3. Vielberg et al., (2018) have used three different methods to estimate the above-mentioned values, which are the multi-step numerical

Table 2. Estimated calibration values for GRACE-A/B accelerometer readouts on January 1, 2005, obtained by wavelet-based method and their estimates according to Bettadpur (2009).

Calibration method	Estimated bias in the flight (along-track) direction (μm/s^2)		Estimated bias in the radial direction (μm/s^2)		Estimated bias in the cross-track direction (μm/s^2)	
	GRACE-A	GRACE-B	GRACE-A	GRACE-B	GRACE-A	GRACE-B
Proposed wavelet-based method	-1.28	-0.51	29.10	10.27	-0.55	-0.81
According to Bettadpur (2009)	-1.21	-0.60	29.34	10.67	-0.56	-0.79

Table 3. Mean values and standard deviations of daily biases of GRACE-A accelerometer readouts during March 2015, in along-track, radial and cross-track directions of the satellite reference frame (SRF), based on the proposed cfc wavelet-based method and the methods applied by Vielberg (2018), as the multi-step numerical estimation (MNE), the dynamic estimation (DE) and the empirical model approach (EMA).

Calibration method		Estimated bias in the flight (along-track) direction (m/s^2)	Estimated bias in the radial direction (m/s^2)	Estimated bias in the cross-track direction (m/s^2)
Proposed wavelet-based method		$-1.2080\times10^{-6} \pm 1.16\times10^{-9}$	$3.009\times10^{-5} \pm 3.25\times10^{-9}$	$-5.7739\times10^{-7} \pm 4.15\times10^{-9}$
According to Vielberg (2018)	MNE	$-1.2655\times10^{-6} \pm 1.15\times10^{-8}$	$2.9149\times10^{-5} \pm 1.41\times10^{-8}$	$-7.4932\times10^{-7} \pm 2.77\times10^{-8}$
	DE	$-1.2686\times10^{-6} \pm 5.95\times10^{-10}$	$2.9149\times10^{-5} \pm 5.09\times10^{-9}$	$-4.9365\times10^{-7} \pm 2.57\times10^{-8}$
	EMA	$-1.1937\times10^{-6} \pm 1.11\times10^{-9}$	$2.9139\times10^{-5} \pm 4.47\times10^{-10}$	$-5.6277\times10^{-7} \pm 9.25\times10^{-10}$

estimation (based on the numerical differentiation of kinematic orbits), the dynamic estimation (using the variational equation approach within a dynamic precise orbit determination) and the empirical model approach (based on modeling the non-gravitational forces acting on the satellite surface). Our proposed wavelet-based scheme has a good adaptation with the above three methods, especially it brings the results closest to the empirical model approach with values that are 0.0143×10^{-6}, 0.0951×10^{-5} and 0.1462×10^{-7} in along-track, radial and cross-track directions, respectively. It is worth considering that $q(t)$ in Eq. (8) should have zero mean. This necessity is violated in terms of some of the constituents of the non-gravitational accelerations; for instance, a continuous drag on the satellite produces non-zero accelerometer readouts results in a constant value of 10^{-7} m/s^2 (Bezděk, 2010). Despite this, it appears that due to the small size of the overall average of the non-gravitational accelerations, the estimated biases have not been affected.

4 Conclusion

In this study, the wavelet-based detrending scheme was developed to calibrate the accelerometers on board GRACE satellites. In the method, there was no need to any reference values for non-gravitational accelerations which could degrade the results due to uncertainties in gravitational field modeling and also some computational problems such as the amplification of noise during the numerical differentiation of satellite positions. Our method also resolved the computational difficulties related to the coordinate transformations in conventional procedures.

Although in theory the least-squares regression is somewhat more accurate than the wavelet-based method of detrending, but for GRACE accelerometer data, the propoased wavelet-based procedure also led to satisfactory results because this method was not dependent on the derivative noise amplification and the reference values for non-gravitational accelerations. This could be the case for all situations which indirect reference values have been obtained as a result of differentiation.

Further investigations could address the studies of (i) tests with other wavelets as the base functions, (ii) calibrating unevenly spaced time series using the wavelet-based method and (iii) revealing non-linear parts in the drift, among others. With the operation of GRACE Follow-on mission, successfully launched on May 22, 2018 (Kornfeld et al., 2019), the proposed method can be tested for its accelerometer readouts too.

References

Andreas, E. L., and Treviño, G., 1997, Using wavelets to detect trends: Journal of Atmospheric and Oceanic Technology, **14**(3), 554-564.

Bettadpur, S., 2009, Recommendation for a-priori bias and scale parameters for Level-1B ACC data (version 2), GRACE tn-02.

Bezděk, A., 2010, Calibration of accelerometers aboard GRACE satellites by comparison with POD-based nongravitational accelerations: Journal of Geodynamics, **50**(5), 410-423.

Case, K., Kruizinga, G., and Wu, S., 2002, GRACE Level 1B Data Product User Handbook: JPL Publication D-22027.

Chen, Y., 2007, Recovery of terrestrial water storage change from low-low satellite-to-satellite tracking: PhD. thesis, The Ohio State University.

Han, S. C., Kim, H., Yeo, I. Y., Yeh, P., Oki, T., Seo, K. W., Alsdorf, D., and Luthcke, S. B., 2009, Dynamics of surface water storage in the Amazon inferred from measurements of inter-satellite distance change: Geophys. Res. Lett., **36**(9), L09403, 1-5.

Kim, J., 2000, Simulation study of a low-low satellite-to-satellite tracking mission: PhD. thesis: University of Texas at Austin.

Kornfeld, R. P., Arnold, B. W., Gross, M. A., Dahya, N. T., Klipstein, W. M., Gath, P. F., and Bettadpur, S., 2019, GRACE-FO: The gravity recovery and climate experiment follow-on mission: Journal of Spacecraft and Rockets, **56**(3), 931-951.

Lemoine, F. G., Luthcke, S. B., Rowlands, D. D., Chinn, D. S., Klosko, S. M., and Cox, C. M., 2007, The use of mascons to resolve time-variable gravity from GRACE, in Tregoning, P., Rizos, C., eds., Dynamic Planet: Springer, Berlin, Heidelberg, 231-236.

Moradi, A., and Sharifi, M. A., 2016, Windowed least-squares spectral analysis of GRACE K-band range rate measurements: Applied Ecology and Environmental Research, **15**(1), 429-437, .doi org/10.15666/aeer/1501_429437.

Švehla, D., and Földváry, L., 2006, From kinematic orbit determination to derivation of satellite velocity and gravity field, in Flury, J., Rummel, R., Reigber, C., Rothacher, M., Boedecker, G., Schreiber, U., eds., Observation of the Earth system from space: Springer, Berlin, Heidelberg, 177-192.

Tangdamrongsub, N., Hwang, C., Shum, C. K., and Wang, L., 2012, Regional surface mass anomalies from GRACE KBR measurements: Application of L-curve regularization and a-priori hydrological knowledge: J. Geophys. Res., Solid Earth, **117**(B11).

Tapley, B. D., Bettadpur, S., Watkins, M., and Reigber, C., 2004, The gravity recovery and climate experiment: Mission overview and early results: Geophys. Res. Lett., **31**(9), 1-4.

Van Helleputte, T., Doornbos, E., and Visser, P., 2009, CHAMP and GRACE accelerometer calibration by GPS-based orbit determination: Advances in Space Research, **43**(12), 1890-1896.

Vielberg, K., Forootan, E., Lück, C., Löcher, A., Kusche, J., and Börger, K., 2018, Comparison of accelerometer data calibration methods used in thermospheric neutral density estimation: Annales Geophysicae, **36**(3), 761-779.

Weigelt, M. L., 2007, Global and local gravity field recovery from satellite-to-satellite tracking: PhD. thesis, University of Calgary, Canada.