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GPR noise reduction by TVD and TVD-EMD

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Abstract

The existence of coherent and incoherent (random) noises including high-frequency electromagnetic inferences in the Ground Penetration Radar (GPR) signals is inevitable. Therefore, the elimination of noise from GPR data before performing any additional analysis is of great importance to increase the accuracy of the interpretations. We apply the Total Variation De-noising (TVD) and Savitzky-Golay (SG) filter on synthetic and real GPR datasets. For a better perception, the same trace of the data is compared after applying the mentioned methods. The results indicate that the TVD method is more effective than the common adaptive filtering in the time domain for reducing noise such as the SG filter which acts as a low-pass filter for smoothing data based on a polynomial least-squares approximation. However, due to the visibility of staircase artifacts using the TVD method, GPR data is first transferred to the Empirical Mode Decomposition (EMD) frame which is useful for non-linear and non-stationary signal processing, and then the TVD method is applied to it. Finally, noise reduction using TVD is compared in the time and EMD domains. The comparison of the outputs shows that the TVD algorithm in the EMD domain, based on the sequential extraction of the energy belonging to the different intrinsic time scales of the signal, provides better noise attenuation than the other algorithms. In addition, TVD-EMD improves the continuity in sections and preserves the event forms and signal forms.

Keywords: De-noising, Ground Penetrating Radar (GPR), Empirical Mode Decomposition (EMD), Savitzky-Golay (SG) filter, Total Variation De-noising (TVD)

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1 Introduction

Ground Penetration Radar (GPR) is commonly used as a technique that employs the high-frequency electromagnetic waves of about 10 MHz to 1 GHz for imaging the shallow subsurface (Jol, 2008). To improve the resolution of the GPR signals and obtain an accurate prediction of underground anomalies, certain preliminary processes are necessary to be applied to the data. The remarkable problem begins with the fact that increasing the signal-to-noise ratio (SNR) while extracting the shape of the GPR trace as close as possible to the convolution of the source wavelet and the earth's reflectivity series, is one of the major challenges that researchers have been confronted in recent years. Several studies have been conducted on the reduction of the GPR noise (Neelamani et al., 2008; Liu et al., 2017; Oskooi et al., 2015). In this article, to estimate the signal of interest, two common methods are applied to the synthetic and real GPR data. The Savitzky-Golay filter which has already proved to have a good performance in de-noising the seismic data based on the estimation of the L_2 norm (Liu et al., 2016), is applied to the GPR data and compared with the TVD method consisting in the L_1 norm.

The SG filter introduced by Savitzky and Golay (1964), can be administered in a series of data points as a low-pass filter to amplify the SNR while preserving the shape of the original signal. Then, the smoothed points are calculated by shifting each data point by the value of its adjusted polynomial (Boudraa et al., 2007; Press and Teukolsky, 1990). Currently, this filter is often applied to digital control system (Kennedy, 2015), ridge detection in image processing (Jose et al., 2013), speech recognition (Krishnan et al., 2013) as well as geophysical data noise reduction (Liu et al., 2016).

TVD is a non-linear filtering in the time domain introduced by Rudin et al. (1992). This method has been generally applied in the treatment of sparse signals as a penalty function in de-noising (Chartrand and Staneva, 2008; Chan et al., 2001) and is defined by the optimization formulation. The result is obtained by minimizing a particular cost function which is non-differentiable (Selesnick, 2012). The proposed TVD algorithm is derived using the maximization-minimization (MM) approach developed by Figueiredo et al. (2006). During the resolution process, the MM algorithm monitors the minimization of the difference between observed and desired data and simultaneously applies the control parameter to the data (Pakmanesh et al., 2018). This method is applied in deconvolution (Moghaddam et al., 2019), reconstruction (Wang et al., 2008) and compressed sensing (Yin et al., 2008).

In GPR dataset, there is a large bandwidth around the center frequency and noise exists on all frequencies. Thus, to solve this problem, the empirical mode decomposition method (EMD) is used to extract the sub-signals. The EMD method is one of the non-stationary and non-linear data processing techniques introduced by Huang et al. (1998) which is based on the decomposition of the energy associated with different intrinsic time scales of the signal ranging from high-frequency modes to lower ones. This method empirically extracts a non-stationary signal into a limited set of almost stationary and oscillatory sub-signals, called intrinsic mode functions (IMF). Each IMF has different frequency components, providing different geological and stratigraphic information (Han and Van der Baan, 2013). At each stage of decomposition, the frequency decreases; therefore, the number of extremes also decreases.

The main purpose of this study is to reduce random noise in GPR data by the improvement of time-domain filtering based on L_1 and L_2 norms in the Empirical Mode Decomposition (EMD) framework. As a result, in this work, the least-squares based SG filter is first applied to the GPR signals, and the calculated SNR is compared to that of the TVD algorithm. After assessing the superiority of applying any of the mentioned filters, each noisy GPR signal in the de-noising procedure is adaptively decomposed in the IMFs by the shifting process in the EMD operator. Next, the optimal method which has better performances to increase the SNR is applied to each IMF. This approach is tested on synthetic and real datasets. The purpose of applying the based shift process and splitting non-linear and non-stationary GPR signals in IMFs is to obtain the almost stationary and linear sub-signals by statistical controls.

2 Theory

The non-stationary signal received from the GPR system, X(t), which is generally contaminated by noise, can be expressed as follows (Oskooi et al., 2015):

$$X(t) = s(t)+n(t) \quad (1)$$

where $s(t)$ is the main signal of the studied object corrupted by noise $n(t)$. The ultimate goal of de-noising is to bring $X(t)$ as close as possible to $s(t)$ while minimizing the effect of $n(t)$.

2-1 Savitzky-Golay filter (SG)

The SG filter is a temporal smoothing filter in which a piecewise adjustment of a polynomial function by least-squares approaches is performed on the signal. The characteristics of this filter have been well discussed in Luo and Ying (2005). In the de-noising process with this method, we consider that the noisy data X(t) is (Sadeghi and Fereidoon, 2018):

$$X(l) = s(l)+n(l) \qquad l = 1, \dots , L \quad (2)$$

which $s(l)$ denotes the l-th sample of $s(t)$. The purpose of the SG filter is to reconstruct $s(t)$ from these samples, so a polynomial $P(i)$ with filter order n ($P(i) = \sum_{k=0}^{n} a_k i^k$, where a_k is the k-th coefficient of the polynomial and $k = 0, \dots, n$), is fitted to smoothing window with the number of data points N=2M+1 to minimize the following MSE:

$$\varepsilon_n = \sum_{i=-M}^{M}(P(i) - x(i))^2 = \sum_{i=-M}^{M}(\sum_{k=0}^{n} a_k i^k - x(i))^2 \quad (3)$$

The filtered signal in the first stage is the value of the polynomial in the central point ($Y(0)$) as follows:

$$Y(0) = P(0) = a_0 \quad (4)$$

The next filtered point is calculated by shifting the window by one and repeating the operation. The ability of this filter to maintain the shape of the signal rather than the other conventional filters is one of the reasons for selecting this method. Moreover, the method is user-friendly and has the high processing speed (Acharya et al., 2016).

2-2 Total-Variation De-noising (TVD)

In total-variation de-noising, it is assumed that the form of the noisy data y(n) is (Selesnick, 2012):

$$y(n) = x(n) + w(n) n = 0, \dots, N-1 \quad (5)$$

where $x(n)$ is a piecewise constant signal and $w(n)$ is white Gaussian noise.

TVD estimates the signal $x(n)$ by solving the following minimization problem (Selesnick, 2012):

$$minimize \sum_{n=0}^{N-1}|y(n) - x(n)|^2 + \lambda \sum_{n=1}^{N-1}|x(n) - x(n-1)| \quad (6)$$

where λ is the regularization parameter that controls the degree of smoothing. This parameter plays a critical role in the de-noising process. When $\lambda=0$, there is no smoothing and the result is the same as minimizing the sum of squares. If λ is too small, then errors in the data will be extremely increased caused by a very noisy approximate solution. If λ is too large, the approximate solution will not be consistent with the data. Thus, the choice of the regularization parameter is critical to achieve just the right amount of noise removal (Selesnick, 2012). It is possible to determine λ by examining a range of values and selecting the optimal value. However, this is only a subjective choice. The methods of Generalized Cross-Validation (Ebrahimi et al., 2017) and L-curve (Hansen, 1999) can be used to obtain an optimum regularization parameter and the number of iterations.

The matrix D is defined as:

$$D=\begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & \ldots & \ldots & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix} \quad (7)$$

The Total Variation of N-point signal $x(n)$ is given by Selsenick (2012):

$$TV(x) := \|Dx\|_1 = \sum_{n=1}^{N-1}|x(n)-x(n-1)| \quad (8)$$

In the above equation, Dx is the first-order difference of an N-point signal of x where D is of size *(N-1)×N*. With the above notation, TVD problem (6) is written compactly as Selsenick (2012):

$$arg\ min\{F(x) = \frac{1}{2} arg\ min\|y-x\|_2^2 + \lambda\|Dx\|_1\} \quad (9)$$

To solve the optimization problem of equation (9), the MM algorithm, which is well described in Selesnick (2012), is used.

In the study of Selsenick (2012), an optimal solution is satisfied by two basic conditions which are as follows:

1. If x is the solution to the TV denoising problem, then it must satisfy:

$$|s(n)| \leq \frac{\lambda}{2} \qquad n = 0, \ldots\ldots, N-1 \quad (10)$$

where *s(n)* is the cumulative sum of the residuals:

$$s(n) := \sum_{k=0}^{n}\big(y(k)-x(k)\big) \quad (11)$$

2. It is further necessary that *x(n)* satisfies:

$$d(n) > 0,\ s(n) = \frac{\lambda}{2}, d(n) < 0,\ s(n) = -\frac{\lambda}{2} \quad (12)$$

$$\mathrm{d(n)} = 0, \qquad \mathrm{s(n)} < -\frac{\lambda}{2}$$

where $d(n) = x(n+1) - x(n)$ is the first-order difference function of x(n).

The condition (12) requires the points to lie on a curve consisting of three line segments (a double-L shape) (Selsenick, 2012).

The number of iterations is another subsidiary parameter for levelling out the cost function and converging the algorithm (Selsenick, 2012). The small iteration number prevents the algorithm to be converged and the large one increases the computation time.

2-3 Empirical Mode Decomposition (EMD)

The EMD method is associated with the decomposition of the given signal *X(t)* into a series of Intrinsic Mode Functions (IMFs), through the selection process, each with a separate time scale. The procedure of signal decomposition with the EMD method is well described by Huang et al. (1998) and Boudraa et al. (2007). The main advantage of EMD is that the basis functions are derived from *X(t)* itself. The decomposition is dependent on the local signal time scale and delivers adaptive basis functions. Each IMF replaces then the detail signals of *X(t)* at a certain scale of the frequency band (Flandrin et al., 2004). The EMD separates the highest frequency oscillation that remains in *X(t)*. So, the measured IMF should satisfy two basic conditions: 1) the number of extrema and the number of zero crossings are either equal or differ at most by one; 2) at any point, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero. Thus, locally, each IMF contains lower frequency oscillations than the one just extracted before (Boudraa et al., 2007). *X(t)* needs to have at least two extrema, one minimum and one maximum to be successfully decomposed in IMFs. The shifting process in this method has two achievements: to remove riding waves and to smooth uneven amplitudes. This method decomposes the given GPR multicomponent signal *X(t)* into L zero-mean IMFs (*i.e.*, $h^{(i)}(t)$) with distinct time scales by statistical methods:

$X(t) = \sum_{i=0}^{L} h^{(i)}(t) + d(t)\ 1 \leq i \leq L$ (13)

where $d(t)$ is a nonzero-mean residual. The shifting process satisfies the following relation:

$$X^{(i)}(t) = \begin{cases} X(t) & ,i = 1 \\ X(t) - \sum_{j=1}^{i-1} h^{(j)}(t) & ,i \geq 2 \end{cases} \quad (14)$$

3 Application of suggested methods on simulated GPR signals

To evaluate the efficiency of the mentioned de-noising methods on artificial GPR signals, a synthetic model was produced using a full-wave solver of Maxwell's equations (GPRMax 2.0). The synthetic model chosen in this study, as shown in Fig. 1-a, is considered to be composed of three layers with different dielectrics and conductivities, where the upper layer is made of concrete (t_1=1m, $\sigma_1 = 0.005\frac{s}{m}, \varepsilon_{r1} = 6$), the middle layer of wet sandy soil (t_2=2m, $\sigma_2 = 0.1\frac{s}{m}, \varepsilon_{r2} = 25$) and the third layer is a saturated sandy soil (t_3=1m, $\sigma_3 = 0.07\frac{s}{m}, \varepsilon_{r3} = 30$). First, to evaluate the performance of each method, Gaussian noise was added to the data. Before any de-noising scheme, the SNR of the noisy data is 2 dB. As it can be seen in Fig. 1-b, the noise has destroyed the data; consequently, the second reflector has been completely disappeared. In the noise reduction process, we applied two de-noising methods (SG filter and TVD). For the first step, we used the SG filter method based on the criterion of least squares (which depends on the L_2 norm) to estimate a real signal from the noisy one. To obtain the best compromise between noise reduction and signal maintenance, the window size and the polynomial degree were selected by trial and error. The result of applying a time-domain SG filter on the synthetic data is represented in Fig. 1-c. The SNR calculated as a measure of the effectiveness of noise reduction after applying the SG filter is 5.11 dB. As seen in Figs. 4-c and 4-d, the SG filter can remove the effect of background noise from the data so that the second reflector is almost visible. However, this technique, as a conventional method, only makes the data more smooth and introduces undesirable events in the signal after de-noising. It indicates that the method cannot distinguish between signal and noise components.

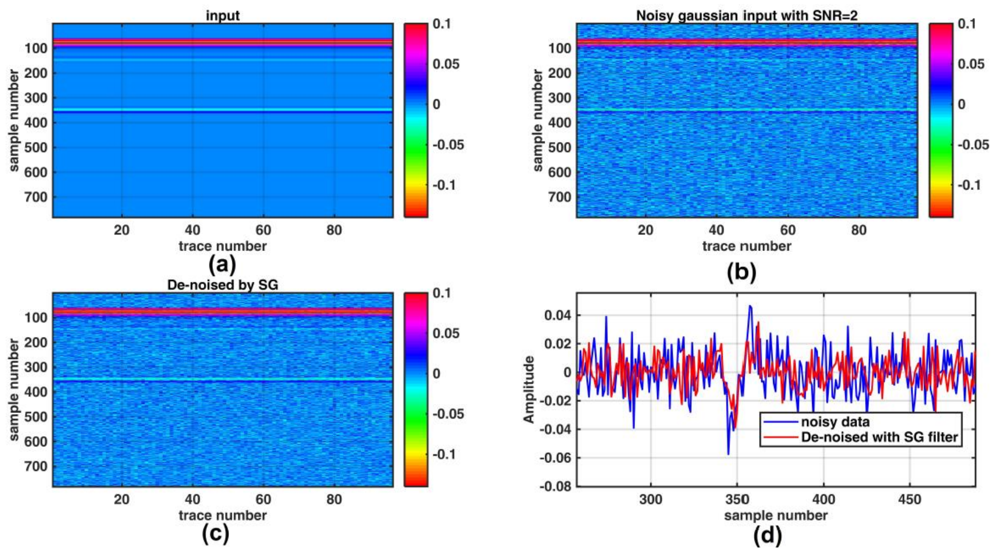


Fig 1. (a) The clean synthetic GPR model. (b) GPR data of the synthetic model contaminated by white Gaussian noise with SNR = 2 dB. (c) De-noised synthetic GPR data after using the SG filter. (d) De-noised synthetic second trace of GPR data after using the SG filter.

In this research, to obtain an appropriate norm for de-noising GPR data, we applied the TVD method which is essentially the L_1 norm of derivatives. According to Selesnick and Chen (2013), TVD is most suitable for constant piecewise signals (Selesnick, 2012); therefore, to modify this method in GPR signal processing, the EMD framework was used to improve the de-noising quality by smoothing the GPR signals. Since random noise in GPR data does not follow the Gaussian distribution, to present the capability of the TVD method, Gaussian and non-Gaussian noisy synthetic GPR data with SNR = 2 were targeted for attenuation. The presence of noise in the GPR data is more evident in higher frequencies. However, these noises are also present at lower frequencies, but it has been proven to give more weight to higher frequencies. Therefore, the higher frequency content modes were selected. The higher modes often have much less random noises because they are closer to the DC signal. As a result, these modes have been ignored in the application of the TVD method in this research. The criteria for ignoring the IMFs are to calculate the averaging of the variance of all IMFs in each trace. In other words, after each decomposing step in the EMD method, the frequency content decreases and turning to the fact that the frequency content of random noises are often in high frequency ranges, the primary IMFs were selected as the most concentrated steps for de-noising of the decomposed GPR data. Experimentally, the IMFs that have a variance of less than half of the average variance of all IMFs, are ignored. By applying the EMD method, the concentration of random noises appears in certain modes, so the assumption of the fragmented signal will be more consistent with the TVD method. This focus makes the proposed method more effective than the time domain. It is very interesting to use different values of D for smoothing, but in the strategy presented in this study, only the capability of the EMD method has been relied on. Selecting of parameters for the fragmental algorithm is largely experimental and expertise-based. Then, replacing this issue by changing the framework can help the researchers in presenting methods industrially. The results of the application of TVD and TVD-EMD on simulated Gaussian and non-Gaussian noisy synthetic GPR data are presented in Figs. 2 and 3, respectively. To compare two approaches, all the input parameters, including the regularization parameter and the number of iterations, were considered to be equal. According to these figures, decomposing each trace to optimum IMFs (Figs. 2-e and 3-e) and then applying the TVD method on the selected IMFs (Figs. 2-g and 3-g), depending on measured average variance (Figs. 2-f and 3-f), indicate the superiority of the TVD-EMD compared to the TVD method. The SNR calculated after using TVD for data corrupted by Gaussian noise is 7.19 dB and for non-Gaussian is 6.38 dB. As shown in Figs. 2-c, 2-f, 3-c and 3-f, the TVD was able to reduce the effect of noise in the data and also the second reflector is better visible. The increase in the amount of SNR compared to the SG filter also confirms this fact. However, the events with lower amplitude were more influenced compared to the clean data of the de-noising process. On the other hand, events of higher amplitude are less influenced. Furthermore, the restoration of the layer border was not well done and the staircase artifacts are visible. It produced a large number of undesirable events resulting from the implementation of filtering. The SNR calculated after the application of TVD-EMD is 8.71 dB for Gaussian and 7.83 dB for non-Gaussian noisy synthetic GPR data. As shown in Figs. 2-d, 2-h, 3-d and 3-h, TVD-EMD was more effective than common TVD in mitigating the effects of noise and restoring borders. Unfavorable events are significantly reduced and the decrease in signal amplitudes was resolved compared to TVD.

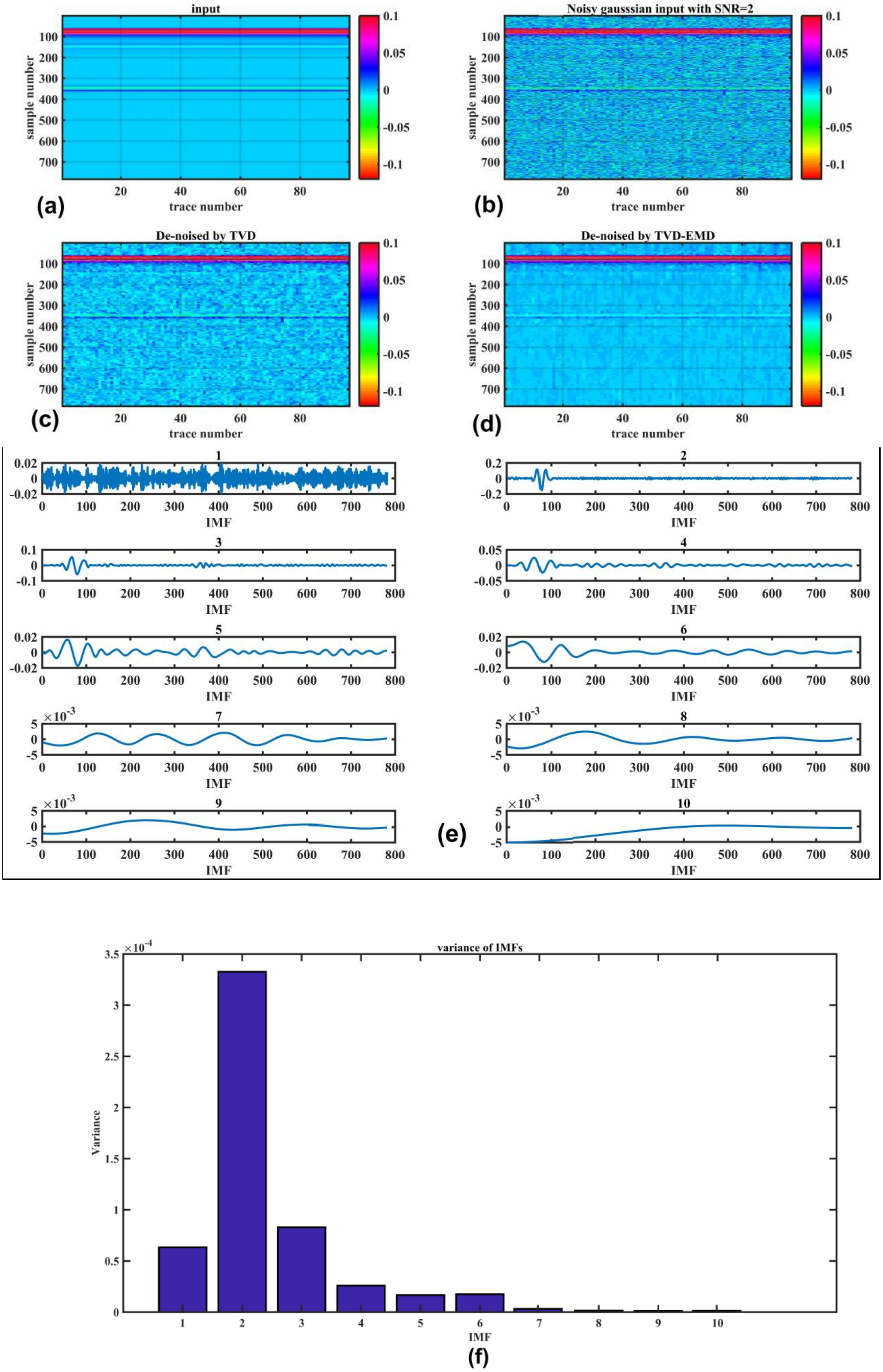
Through the representation of the power spectrum density, it is possible to better see the elimination of noise and signal as a fundamental quantitative and qualitative challenge. Because applying many filters may cause some frequency ranges to be omitted, the most important result that can be obtained from power spectrum is the analysis of the frequency response of the applied filter. Therefore, it is very necessary to study the power spectrum in maintaining the signal. Another point that should be mentioned is the issue of noise attenuation. The amount of noise attenuation that corresponds to the selection of applied filter parameters, can be instrumental in determining the effective parameters. Furthermore, changes in the signal are not visually visible in the time domain; accordingly, the power spectrum can be a practical tool for comparing methods in the frequency domain.

Due to the proximity of the SNR calculated in the qualitative and quantitative evaluations of the de-noising and signal maintenance methods, the power spectral densities of the noisy and de-noised data are represented in Figs. 2-i and 3-i.

At lower frequencies, the two methods of TVD and TVD-EMD have almost the same downward trend and are not successful. However, as the frequency range increases, the trend of the TVD-EMD coincides with the true signal (red line) which indicates the signal holding power. In addition, at higher frequencies, the maximum noise reduction is obtained by the TVD-EMD method. It should be also mentioned that the implementation of this method has more computational cost than the TVD method. Also, when there is a large amount of random noise in the GPR data, TVD-EMD is recommended in terms of signal retention and noise attenuation. On the other hand, the best results are obtained with optimum regularization parameter λ and the number of iterations.

None of the proposed methods can eliminate the noise. This is justified by the fact that the presence of noise in geophysical data leads to the stability of inversion processes in some operators such as deconvolution. Every method, regardless of its type and the framework in which it is implemented, has side effects on the signal. It is also important to note that no method is ideal and the noise behavior is very complex and unpredictable. The purpose of this study is to show that the EMD method, despite its computational simplicity, can be a platform for implementing other noise reduction methods in geophysical studies. Of course, these results may not be the best, but this style of study has not yet been implemented in geophysical topics especially in GPR noise reduction studies and is the first of its kind. Therefore, this study is an introduction to the optimization of inversion-based filters in the EMD framework.

To satisfy the optimal solution for the TVD method in which the input parameters of TVD-EMD are the same, two basic conditions including the cumulative sum of the residuals (eq. 10) and the first-order difference function of output (eq. 12) are depicted as scatter plots in Figs. 2-j and 2-k and Figs. 3-j and 3-k. It can be seen that the cumulative sum s(n) is bounded by lambda and the TVD solutions for each Gaussian and non-Gaussian data require the points to lie on the graph of the sign functions. The selected lambda is the reliable value lied on a double-L curve.


input
Noisy gausssian input with SNR=2
De-noised by TVD
De-noised by TVD-EMD
sample number
trace number
(a)
(b)
(c)
(d)
IMF
(e)
variance of IMFs
Variance
IMF
(f)

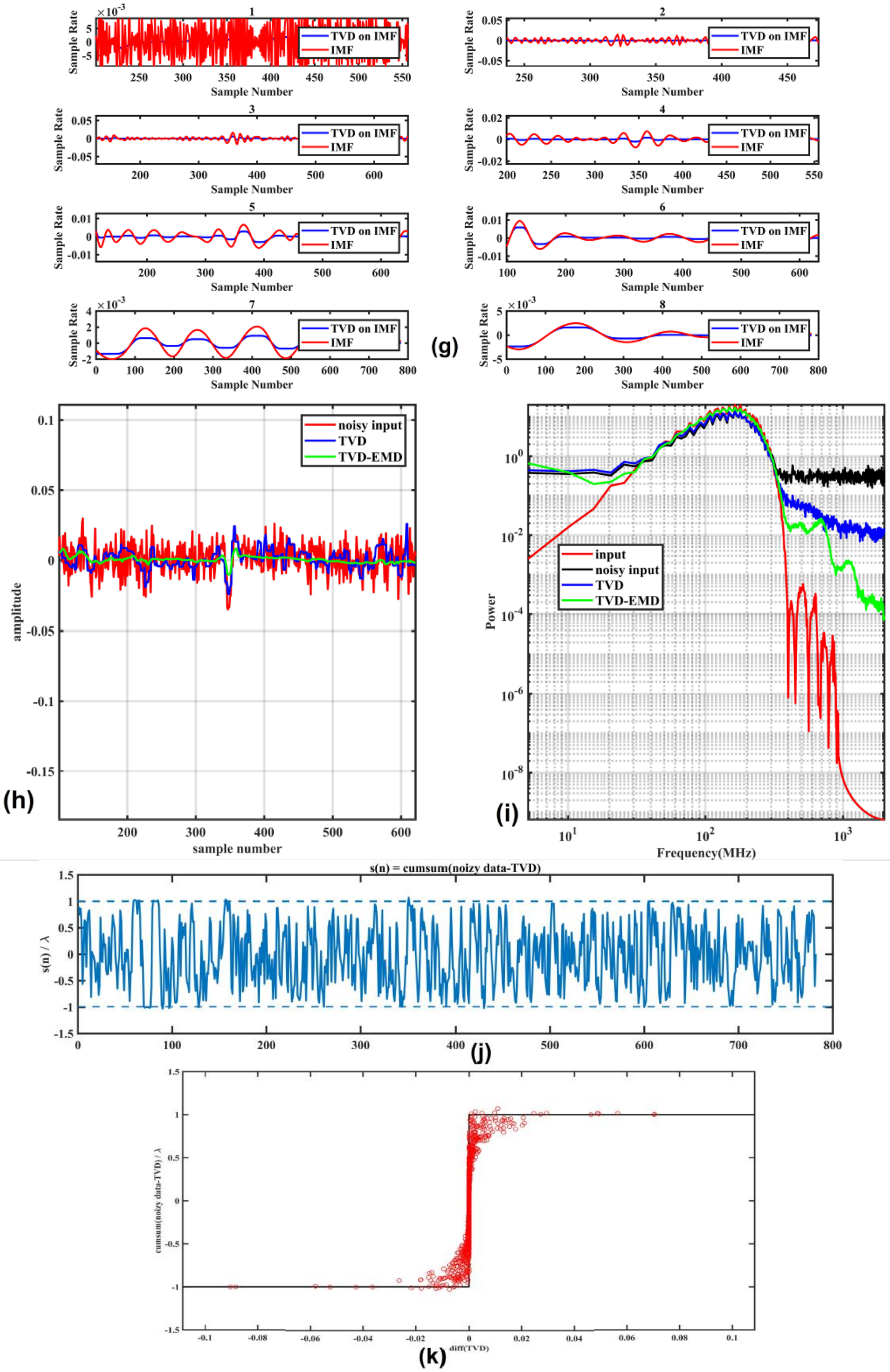
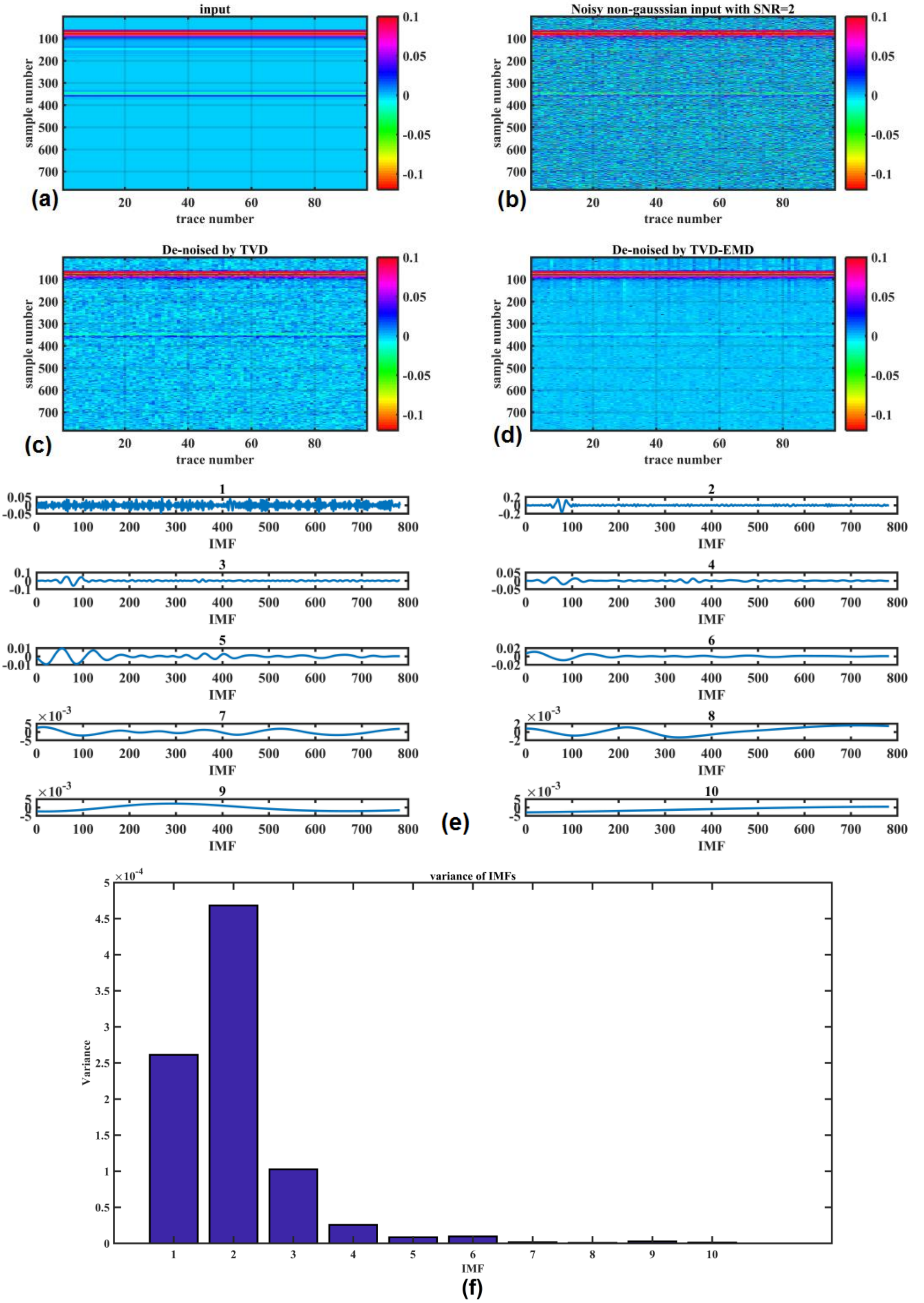
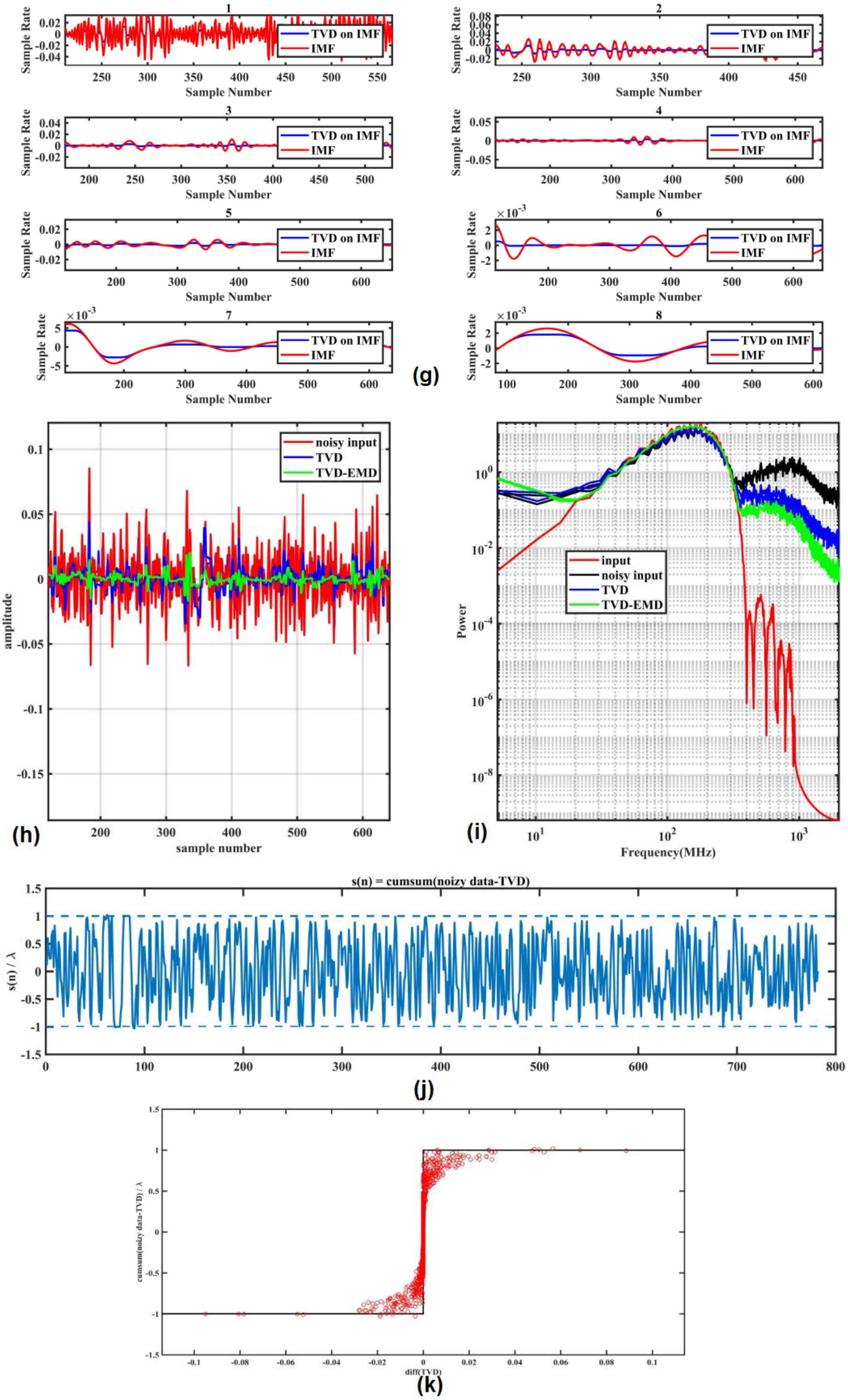


Fig 2. (a) The clean synthetic GPR model. (b) Gaussian noisy input with SNR=2. (c) De-noised by TVD. (d)

De-noised by TVD-EMD. (e) Decomposing of each trace to IMFs. (f) The variance of each IMF. (g) TVD applied on selected IMFs of one of the traces corrupted by Gaussian noise. (h) Comparison of TVD and TVD-EMD on one trace. (i) The power spectrum of GPR data after applying TVD and TVD-EMD with λ=0.03. (j) The cumulative sum s(n) bounded by lambda. (k) Scatter plot of the first-order difference function of output.





1
2
3
4
5
6
7
8
Sample Rate
Sample Number
TVD on IMF
IMF
(g)
noisy input
TVD
TVD-EMD
amplitude
sample number
(h)
input
noisy input
TVD
TVD-EMD
Power
Frequency(MHz)
(i)
s(n) = cumsum(noizy data-TVD)
s(n) / λ
(j)
cumsum(noizy data-TVD) / λ
diff(TVD)
(k)

Fig 3. (a) The clean synthetic GPR model. (b) Non-Gaussian noisy input with SNR=2. (c) De-noised by TVD. (d) De-noised by TVD-EMD. (e) Decomposing of each trace to IMFs. (f) The variance of each IMF. (g) TVD applied on selected IMFs of one of the traces corrupted by non-Gaussian noise. (h) Comparison of TVD and TVD-EMD on one trace. (i) The power spectrum of GPR data after applying TVD and TVD-EMD with λ=0.03. (j) The cumulative sum s(n) bounded by lambda. (k) Scatter plot of the first-order difference function of output.

4 Application of suggested methods on real GPR data

To evaluate the functionality of the proposed de-noising method in the case of real data, we process a set of real GPR data collected using a 100 MHz antenna from the GSSI (Geophysical Survey System, Inc.) company. The actual data after applying the gain as an amplitude recovery function on the data is represented in Figs. 4 and 5-a. Concerning these figures, the data quality is poor and the data is full of noise from various unknown sources.

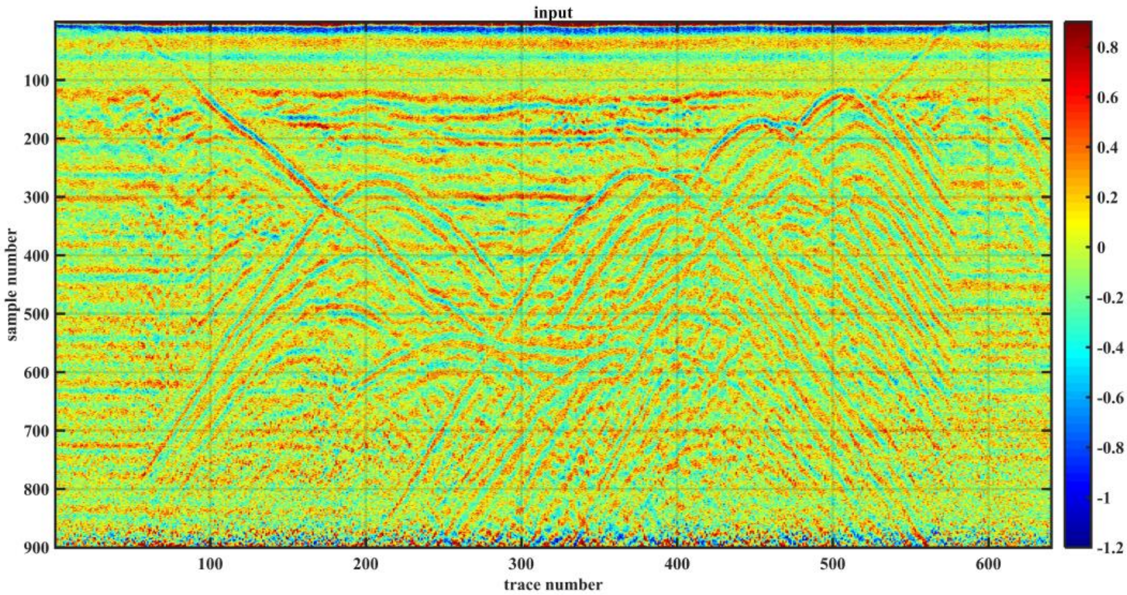
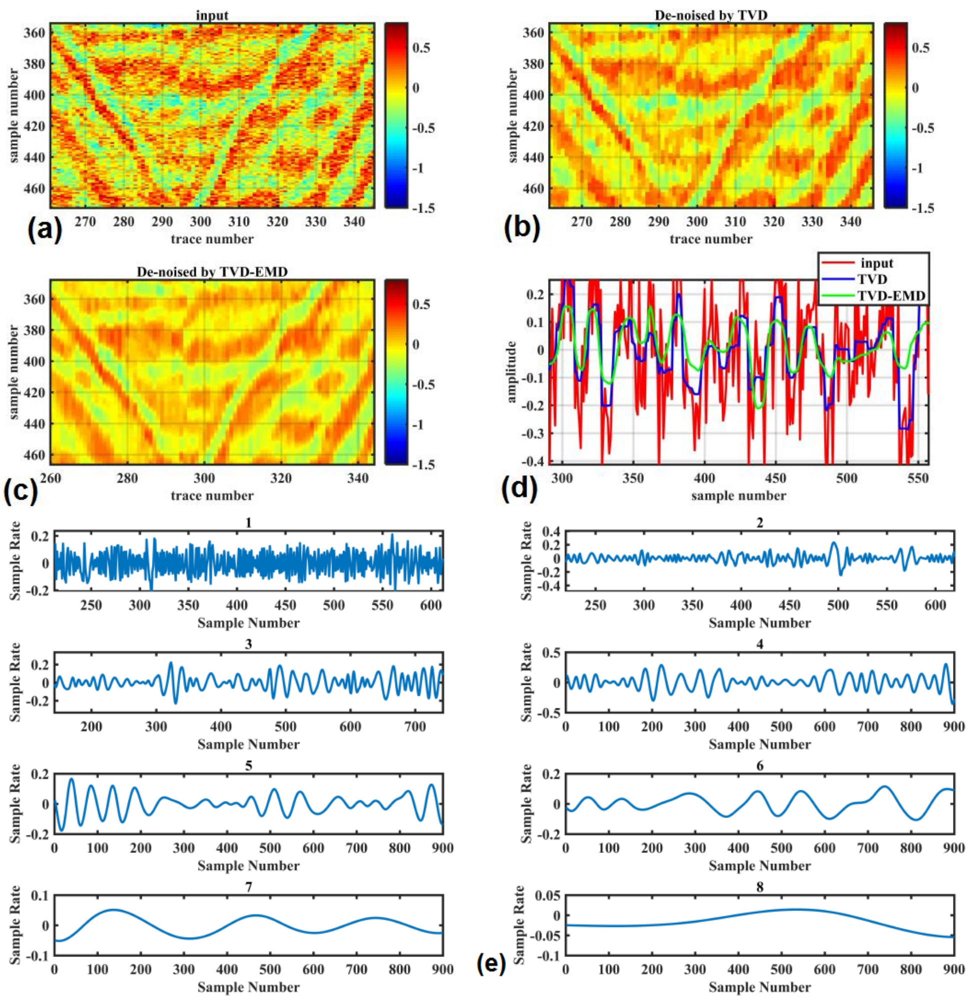


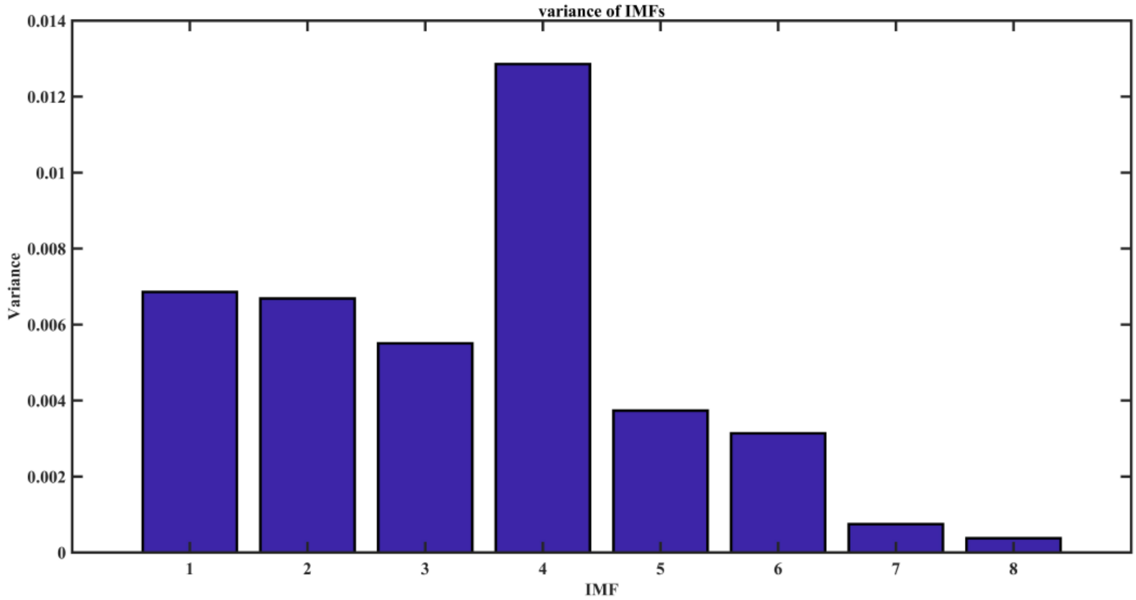
Fig 4. The real GPR dataset using a 100 MHz antenna.

The resulting de-noised data after application of the TVD and TVD-EMD methods with magnification are represented in Figs. 5-b and 5-c. Furthermore, to better demonstrate the capability of each method, the second trace of the real data as an arbitrary trace before and after de-noising is extracted (Fig. 5-d). According to Fig. 5-b, after applying TVD, it can be seen that the noise part has been slightly removed; therefore, the boundaries of many layers have been distorted and the events in the final tales of the data do not appear precisely. On the other hand, the structures of lower amplitudes were more affected by de-noising and the structures of higher amplitudes were better specified. In each event, the restoration of the boundaries has not been well resolved. The reason is that the output of the TVD filter is obtained using the numerical convex algorithm which is very effective in piecewise constant signals according to the Selesnick (2012).

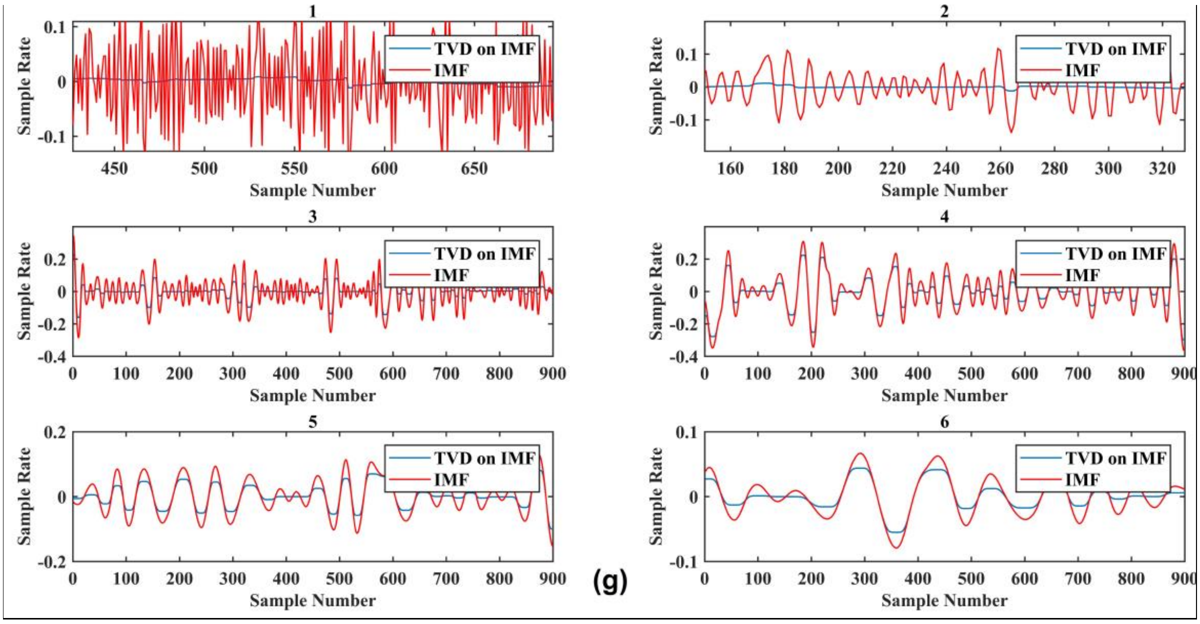
In this approach, the TVD has been modified in the EMD framework to be more applicable in the non-stationary GPR signals. Therefore, the reliable results can be achieved after applying the EMD method on each trace (Fig. 5-e) and using experimental strategy including ignoring of each IMF smaller than half of the average variance (Fig. 5-f). The results of the 60 traces are shown in Fig. 6

which indicates the superiority of the TVD-EMD method compared to the TVD algorithm. By using this filter, the boundary of the layers is well detected and a logical smoothness on the path of each trace is seen. Moreover, the structures with low and high amplitudes are well detectable compared to the results obtained when the temporal filter (TVD) is applied. According to the results illustrated in Fig. 6-c, it can be concluded that by using the TVD-EMD method, the coherence and continuity of the layers are increased and the details of the data are better demonstrated by this method. When using the TVD-EMD method, the noise was remarkably attenuated, the adverse events decreased considerably and the decrease in signal amplitudes was also resolved compared to those of the TVD method. The boundaries of layers marked in Fig. 6-c can confirm the high capacity of the TVD-EMD algorithm compared to the TVD method. The power spectral densities of the noisy and de-noised data after application of the TVD and TVD-EMD methods have been represented in Fig. 6-d. From this figure, it is revealed that the TVD method has only succeeded in eliminating high-frequency noise in the context of the whole time domain, and at low and intermediate frequencies, it treats like a true signal with unwanted structures or artifacts which are observed in Fig. 5-b. However, by examining the power spectrum of TVD-EMD, the appropriate effect of this filter can be seen. At the lowest frequencies, some noises have been added to the data, but in the middle and high frequencies, noise reduction has been done well.





(f)



(g)

Fig 5. (a) The real GPR data after applying AEC gain. (b) De-noised by TVD. (c) De-noised by TVD-EMD. (d) De-noised second trace by TVD and TVD-EMD with λ=1.5. (e) Decomposing of each trace to IMFs. (f) The variance of each IMF. (g) TVD applied on selected IMFs of one of the traces corrupted by noise.

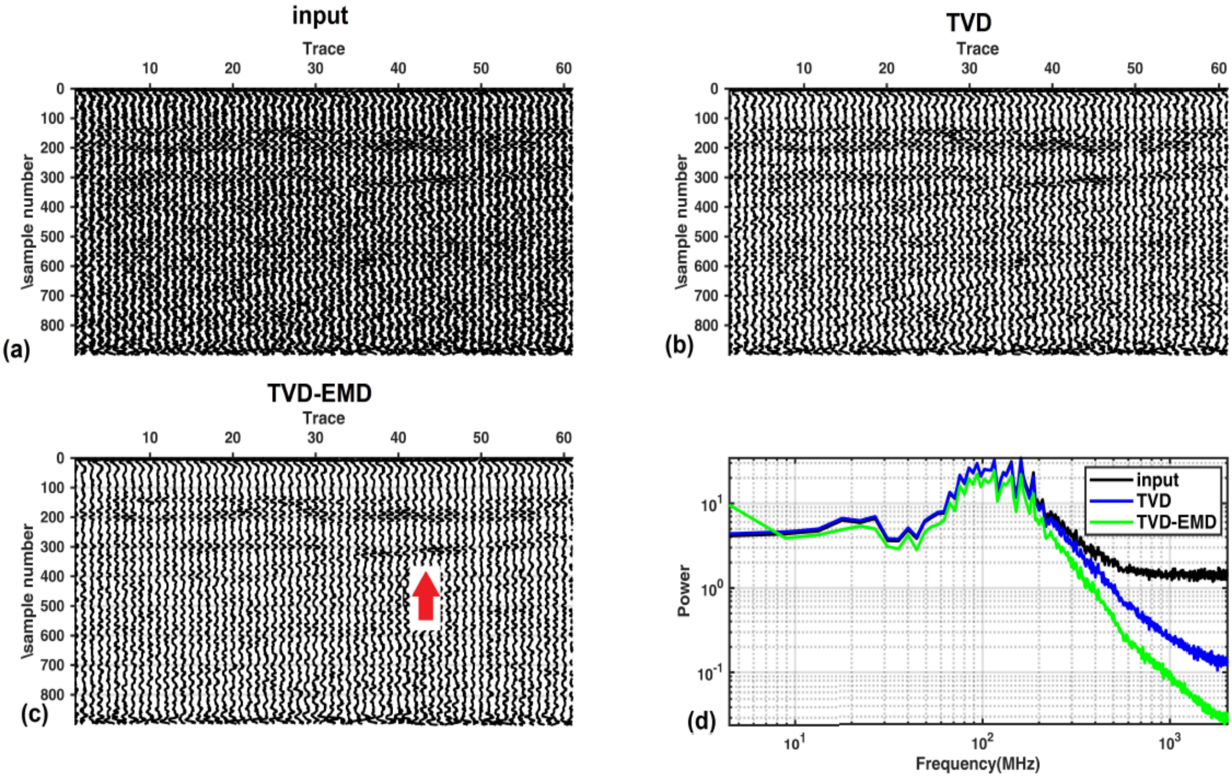


Fig 6. (a) 60 traces of the real GPR data. (b) De-noised by the TVD method. (c) De-noised by the TVD-EMD method. The red flash symbol implies the capability of the TVD-EMD in tracking the path of the layers. (d) Power spectral densities of data by TVD and TVD-EMD.

5 Conclusions

This article focuses on the reduction of random noises in GPR data by applying total variation de-noising based on the empirical mode decomposition framework. First, the TVD method and the SG filter were applied to the synthetic data created from a three-layer model with different physical properties. Adding white Gaussian noise to the synthetic data gave an SNR = 2 dB. By applying the SG filter to the data, the SNR increased to 5.11 dB. After applying the TVD, this amount increased to 7.21 dB. Depending on the fact that the increase in SNR is representative of the efficiency of the de-noising method, one can conclude from SNR and 1D diagram calculated from the two methods on synthetic data that the TVD method has provided better results than the time domain adaptive filter based on the SG filter. In addition, the TVD method was applied to noisy data both in standard form and as part of empirical mode decomposition on noisy Gaussian and non-Gaussian noisy synthetic and real GPR data. By comparing the results, it can be seen that the two methods of TVD and TVD-EMD can considerably mitigate the noise effects. However, the implementation of TVD in the time domain creates artificial noises due to filtering. By decomposing each trace into a series of IMFs, and applying TVD on each IMF, the process of de-noising GPR data is improved. We conclude that the TVD-EMD method preserves events and signals. It decreases staircase events and improves continuity in sections.

For better de-noising of the GPR signal, we plan to study the improvement of the SG filter algorithm in each GPR IMF. Owing to strong theory, application of variational mode decomposition (VMD) instead of the EMD method is also on our agenda for future studies.

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