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The detection of 11th of March 2011 Tohoku's TEC seismo-ionospheric anomalies using the Singular Value Thresholding (SVT) method

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Abstract

The Total Electron Content (TEC) measured by the Global Positioning System (GPS) is useful for registering the pre-earthquake ionospheric anomalies appearing before a large earthquake. In this paper the TEC value was predicted using the singular value thresholding (SVT) method. Also, the anomaly is detected utilizing this predicted value and the definition of the threshold value, leading to the use of the anomaly as a precursor. The SVT is used in the matrix completion problem, namely the accurate recovery of a matrix from a nearly minimal set of entries. In this study, the SVT has been applied to the ionospheric TEC of the global ionosphere maps (GIM) data on a powerful earthquake in Tohoku on the 11th of March in 2011. In this method, the two-hour TEC observations of this region are converted into a matrix for several consecutive days before and after the occurrence of an earthquake. In this matrix the rows and the columns represent the days and the sequential hours, respectively. The prediction of the non-linear time series is formulated as a method for solving the low-rank recovery problem. Results indicate that under suitable conditions the TEC values can be estimated properly in the aforementioned days and hours by solving a simple optimization problem. In order to show the efficiency of this method in predicting the time series, the results obtained from this research were compared with those from other researches.

Keywords: Singular Value Thresholding, anomaly detection, TEC, earthquake, ionosphere

1 Introduction

The modern methods to predict earthquakes are based on the complex analyses of the spatial and temporal earthquake precursors. In order to make accurate prediction, it is required that various prediction methods be utilized simultaneously. One of such methods is

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to investigate the earthquake precursors in the ionosphere. Since the changes in different ionosphere parameters were detected in the Alaska earthquake in 1964, numerous researchers in the field have focused upon the ionospheric precursors. The abnormal changes in the various ionospheric parameters arising from the earthquake have been investigated using different methods. One of these changes involves the detection of the ionospheric disturbances by observing the “total electron content” (TEC) with regard to the related seismic activities. Liu et al. (2004) studied the ionospheric precursor parameters of TEC data on twenty catastrophic earthquakes in Taiwan using statistical evidence from 1 to 5 days before the earthquake. Devi et al. (2008) showed that the electric field produced by the earthquake has a great effect upon the formation of TEC profiles in the anomaly in the northern equatorial regions. Zhao et al. (2008) detected the abnormal TEC changes three days before Sichuan's earthquake on the 12th of May in 2008. Akhoondzadeh (2013) examined the abnormal TEC changes in three catastrophic earthquakes using SVM (Singular Value Method).

Should the ionospheric disturbances be meaningful and systematic, they can be regarded as short-term precursors taking place a few hours or days before an earthquake. It must be pointed out that it cannot necessarily be expected that the entire earthquake precursors occur for every earthquake. Since each precursor is not an accurate indicator to predict the earthquake, it is essential that various precursors be extracted and combined. In the case of the ionosphere, the ionospheric anomalies could be attributed to the occurrence of catastrophic earthquakes, provided that the geomagnetic and solar disturbances do not exist. These anomalies tend to occur in the D, E and F layers of the ionosphere and could be observed one to ten days before the earthquake and even a few days after the earthquake (Hayakawa and Molchanov, 2002; Pulinets and Boyarchuk, 2004). With regard to the mechanism of these anomalies, the electric fields produced prior to the earthquake cause the electrons in the F layer to penetrate into the lower layers, producing anomalies in the ionospheric parameters. Alternatively, prior to the earthquake the thin layer of the particles, which is formed by the radiation of ions from the Earth, plays a major role in transferring the electric field to the upper atmosphere layers and then to the ionosphere. The vertical electron field on the Earth is made into the electric field perpendicular to the geomagnetic field, resulting in the disturbances of the distribution of atmosphere plasma in the region where an earthquake takes place (Namgaladze et al., 2009). Considering the desirable results obtained by the researches in the field of the anomaly observation prior to an earthquake in TEC changes, in this research the variations in this parameter have been investigated before and after the days of the sample earthquake using the “singular value thresholding” (SVT) method. The prediction of the nonlinear time series is formulated as a method for solving low-rank recovery problem.

2 Methodology

Beginning with the works of Candes and Reacht (2009), Candes and Tao (2009) as well as Candes and Plan (2009), the SVT has widely been investigated in recent years. The SVT refers to the recovery of a low-rank matrix from the sampling of its entries. It is commonly used whenever the partially filled-out surveys are collected and many missing entries are inferred. Most low-rank matrices can be recovered entirely from most sets of the sampled entries by minimizing a nuclear norm function subject to the linear constraints. In this study the TEC values obtained from the model have been placed in a matrix in such a manner that the columns and the rows represent the TEC values in the sequential daily hours and in the monthly days, respectively. By forming this matrix, the TEC values can be predicted easily in the intended hours and days through assuming its values as zeros. It can be shown that should the difference between the predicted value using the SVT method and the observed value exceed that of the pre-defined threshold, the observed unusual precursor value in the absence of non-seismic effective parameters could be regarded as an earthquake anomaly. Given the unknown matrix M∈$R^{m\times n}$, the low-rank matrix recovery problems, such as the matrix completion problem, can be formulated by solving the optimization problem:

$$\begin{cases} Minimiz & \|x\|_* \\ S.t & X_{ij} = M_{ij} \quad (i,j) \in \Omega, \end{cases} \tag{1}$$

where $\Omega \subset \{1,2,\ldots,m\} \times \{1,2,\ldots,n\}$ is a random subset of the indices of its sampled entries and $\|.\|_*$ is a nuclear norm (i.e the sum of the singular values). The minimization of $\|x\|_*$ is a convex relation to minimize the rank of X. It is noteworthy that this algorithm fails when the training data is too sparse. Numerous algorithms have contributed to solving this optimization problem. The optimization problem (1) can be written as a Lagrange multiplier algorithm known as Uzawas' algorithm.

$$\begin{cases} Min & c\|\Psi\| + \frac{1}{2}\|X\|_F^2 \\ S.t & P_\Omega(X) = P_\Omega(M) \end{cases} \tag{2}$$

where P_Ω is the orthogonal projector onto the span of the matrices vanishing outside Ω so that the $(i,j)^{th}$ component of $P_\Omega(X)$ can be equal to X_{ij} if $(i,j) \in \Omega$ and zero otherwise, and X is the optimization variable and $\|.\|_F^2$ is the matrix Frobenius norm. With the given $\tau > 0$ and due to starting with $Y^0 = 0$ the algorithm is defined as (Cai et al., 2010):

$$\begin{aligned} X^k &= D_\tau(Y_{k-1}) \\ Y^k &= Y^{k-1} + \sigma_K P_\Omega(M^T - X^K) \end{aligned} \tag{3}$$

so that convergence criterion can be achieved. In this equation D_τ is the SVT operator defined as (Cai et al., 2010):

$$D_\tau(Y) = U \begin{bmatrix} (\sigma_1 - \tau)_+ & & \\ & \ddots & \\ & & (\sigma_s - \tau)_+ \end{bmatrix} V^T \tag{4}$$

where α_+ is the positive part of α, the columns of U are the left singular vectors , V^T has rows that are the right singular vectors, σ_i is the singular value of $Y \in R^{m\times n}$ and $\{\sigma_k\}_{k\geq 1}$ is the scalar step size. The algorithm is terminated when the relative error:

$$\frac{\|P_{\Omega_T}(X - M)\|_F}{\|P_\Omega(M)\|_F} \leq \varepsilon \tag{5}$$

falls below a predetermined tolerance $\varepsilon > 0$.

3 Implementation

In this paper the effect of Tohoku's earthquake upon the TEC value obtained from "global ionosphere maps" (GIM) is examined. This earthquake with a magnitude of $M_w = 9$ occurred near the northeast coast of Honshu in Japan on March 11th, 2011 at 14:46:23 local time. The characteristics of the Tohoku's earthquake accompanied by its main foreshocks' information are given in Table 1.

In order to examine this earthquake, it is required that the ionosphere be investigated under geomagnetic and solar conditions. For this purpose, the indices D_{st}, K_p and $F_{10.7}$ accessed through NOAA (http://spider.ngdc.noaa.gov/) have been used (Mayaud, 1980).

As mentioned before, the solar radiations have effects on the Earth ionosphere and complicate the ionospheric variations made by the earthquake. The magnetic storms can intensify or reduce the ionospheric variations made by the earthquake. Hence, it is essential that the solar and geomagnetic activities be explored. The Figures 1a and 1b display the magnetic conditions before and after Tohoku's earthquake.

According to Figure 1a, the K_p index shows the magnetic activities 21 and 35 days prior to the earthquake. The Kp index is a global indicator that reflects the geomagnetic activity level in a given three-hour time interval. It is derived using globally distributed (in longitude) sub-auroral ground-based magnetometers. Kp indices range from 0 to 9. A Kp index can be interpreted as follows: a value less than 4 indicates a low level of ionospheric activity, a value of 4 indicates a moderate ionospheric level, and a value of 5 or greater indicates a storm level of geomagnetic activity; however such values must be interpreted carefully since they reflect the global level of geomagnetic activity but do not specify the region of high activity.

Table 1. The characteristics of Tohoku's earthquake and its main foreshocks (reported by http://earthquake.usgs.gov/).

Date	Time (LT)	Geographic Latitude, longitude	Magnitude (MW)	Focal depth (km)
9 March 2011	11:45:20.33	$38.44^\circ N, 142.84^\circ E$	7.3	32
10 March 2011	03:16:16.44	$38.31^\circ N, 142.43^\circ E$	6	22
10 March 2011	06:24:01:68	$38.30^\circ N, 142.81^\circ E$	6.4	15
11 March 2011	14:46:24:12	$38.32^\circ N, 142.37^\circ E$	9	29

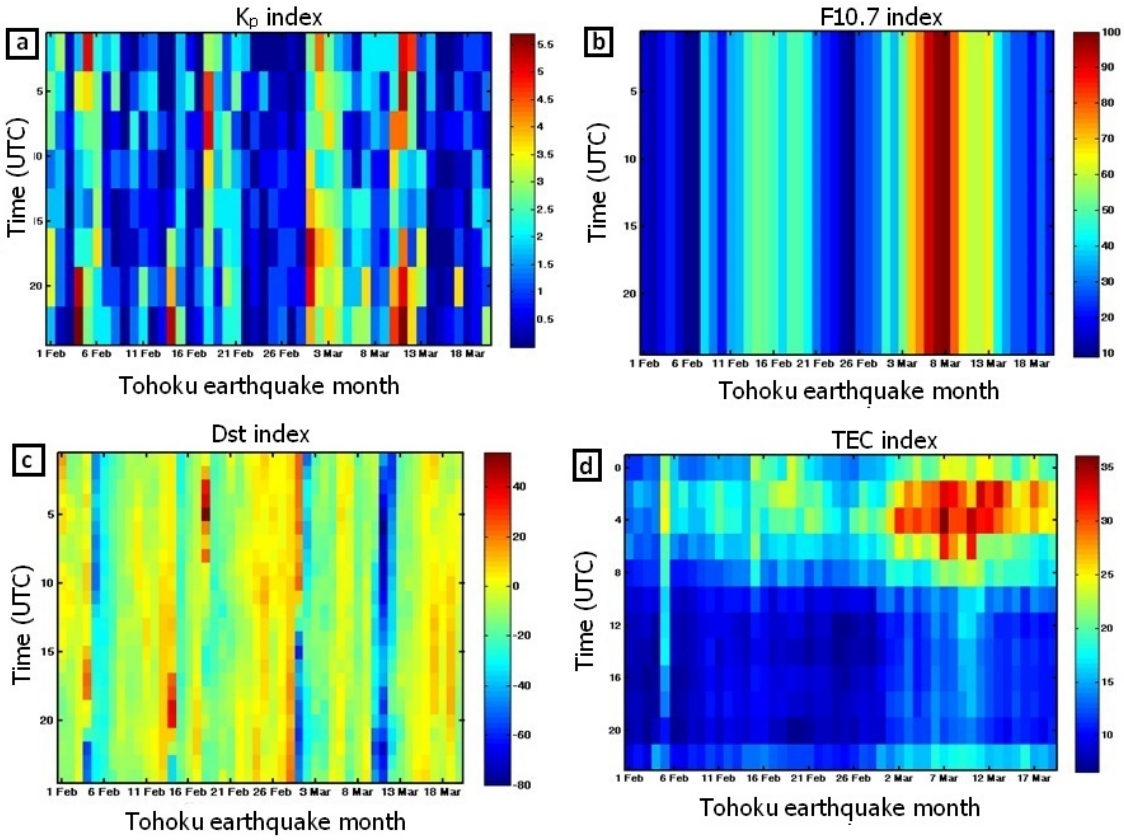

Figure 1. (a), (b) and (c) show, respectively, the variations of Kp, F10.7 and Dst geomagnetic and solar indices during the period of 1 February to 21 March 2011. (d) TEC variations. An asterisk indicates the earthquake time. The x-axis represents Tohoku's earthquake month. The y-axis represents the universal time coordinate.

Figure 1b shows the variations of the solar radio flux (F10.7) from 1 February to 21 March 2011. F10.7 is often expressed in solar flux units (SFU). The F10.7 value gradually increases from about 14 days before the earthquake and reaches the maximum value of 164.30 (SFU) on 8 March 2011, which is 3 days before the event. A parameter known as the solar flux, is used as the basic index of solar activity, and to specify the surface or radiation being received from the sun. The solar flux is measured in solar flux units (SFU) and is the value of radio noise or flux that is emitted at a wavelength of 10.7 cm. The Penticton Radio Observatory in British Columbia, Canada records this factor daily. The solar flux is linked to the value of ionization and the electron condensation in the F2 region. Finally it gives an excellent symptom of conditions for long-distance connection. The F10.7 index indicates a measure of diffuse, non-glittering heating of the coronal plasma confined by magnetic fields over active regions. This index is a great gnomon of totally solar activity levels. F10.7 is linked to the radio energy of slowly changing severity emitted by the sun that results from atmospheric layers high in the sun’s chromosphere and low in its corona. Although the TEC is subjected to an explicit seasonal modulation, the generic manner of TEC is extremely associated with F10.7.

Figure 1c demonstrates the variation in the D_{st} index. The Dst (Disturbance storm time) demonstrates only specified section of the geomagnetic activity at low and high latitudes, respectively. This index exists at hourly or smaller intervals and thus presents improved temporal resolution compared to the 3 hourly K-derived indices. The Dst index captures the equatorial curl current variations. Based upon this figure, the unusual D_{st} values reveal the severe geomagnetic activity from 7-10 and 32-35 before the earthquake. Thus, the resulting TEC variations must not be taken into account as precursors.

Figure 1d illustrates the TEC values obtained from GIM near the earthquake epicenter from February 1 to March 21. As can be seen, some disturbances are observed around the earthquake day.

It is necessary that the variation limits be defined in order to detect the anomalies. In this study, the anomalies have been detected utilizing the mean and standard deviation parameters. For this purpose, the mean and standard deviation have been used to define the upper and lower limits, thereby detecting the earthquake anomalies, as compared with other variations. These limits are determined based upon the following relations:

$$x_{high} = \mu + k\sigma, \tag{6}$$

$$x_{low} = \mu - k\sigma, \tag{7}$$

$$x_{low} < x < x_{high} \Rightarrow -k < \frac{x-\mu}{\sigma} < k, \tag{8}$$

$$Dx = \frac{x-\mu}{\sigma}. \tag{9}$$

x, μ, x_{high}, x_{low}, σ and D_x are the parameter value, mean value, upper limit, lower limit, standard deviation and deviation of parameter value respect to mean value, respectively. In accordance with the above relations, should the D_x absolute value be greater than k, the x intended behavior will be detected in an abnormal manner. Also, according to these relations, $p = \pm 100(|Dx| - k) / k$ indicates the percentage of the parameter deviation from the normal condition. The k parameter value must be proportional to the earthquake magnitude, which is set empirically based on previous researches.

In order to apply the recommended algorithm, the TEC values have been placed in a matrix as mentioned before. In Figure 2 the blue and red curves stand for the observed and predicted values, respectively.

Figure 3 shows the difference between these two values. This figure also demonstrates the upper and lower limits. As can be seen in this figure, the difference between the TEC observed values and the predicted values has exceeded the defined boundaries 5 days before the earthquake at 06:00,10:00,14:00 and 18:00; 4 days before the earthquake at 02:00,04:00 and 18:00; 3 days before the earthquake at 08:00,12:00,14:00,16:00 and 24:00; 2 days before the earthquake at 02:00,06:00,14:00,16:00,18:00,20:00 and 22:00; 1 day before the earthquake at 06:00,10:00 and 22:00; and finally the earthquake day at 04:00,18:00 and 20:00.

Due to the fact that the geomagnetic activity was not quiet during the days around the studied earthquake day, the observed ionospheric disturbances cannot be interpreted independently of the solar-terrestrial events. Hence, investigating the solar-terrestrial coupling mechanism during seismic activity is necessary to comprehend the complex relationship between the TEC ionospheric anomalies and the occurrence of large earthquakes.

To do this, the TEC anomaly can be obtained under the following conditions:

$$\left|DTEC\right| > 2.5, K_p < 2.5, D_{st} > -20(nT) \tag{10}$$

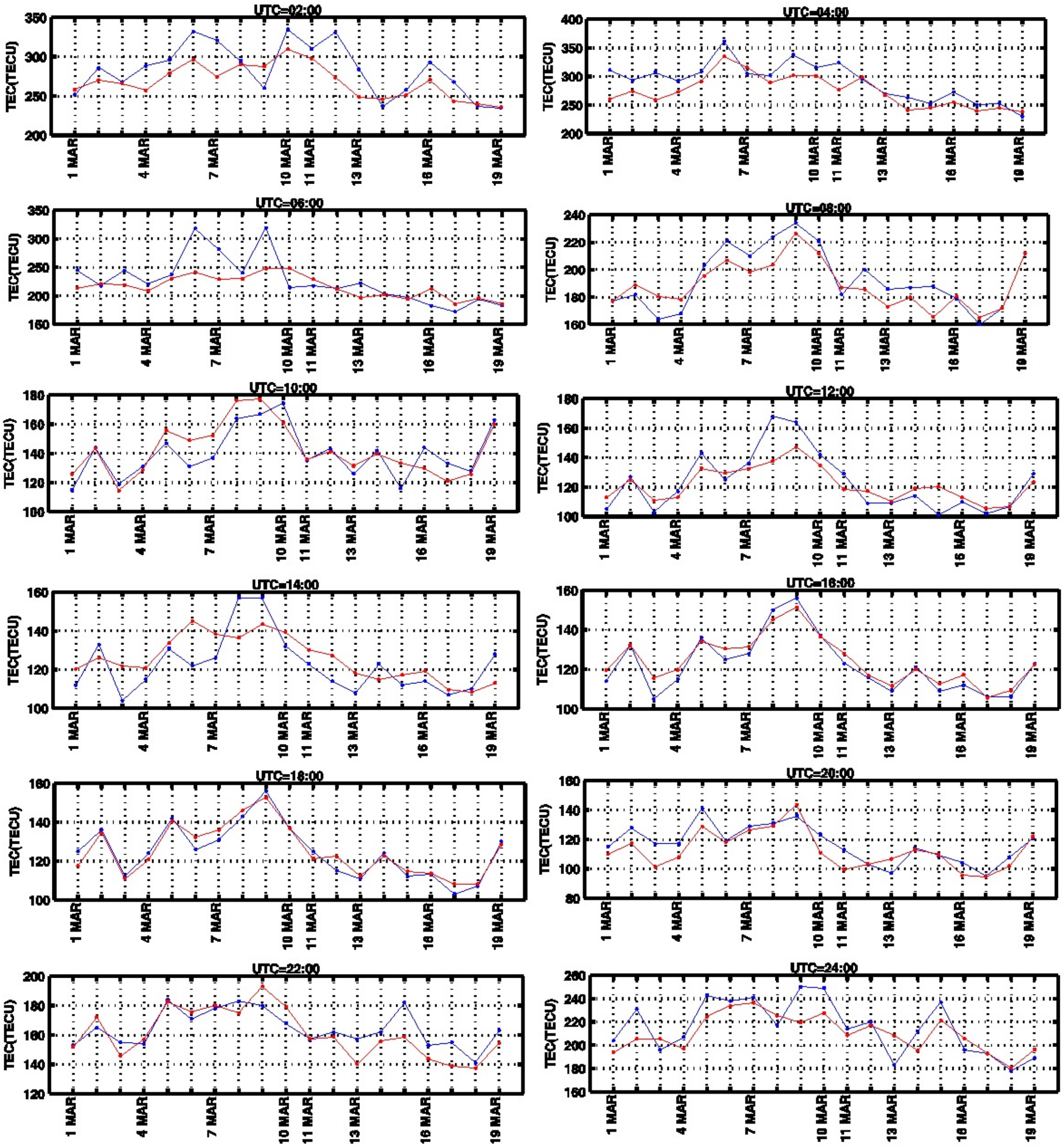

Figure 2. Variations of the observed (blue curve) and predicted (red curve) TEC values on days selected as testing set at different universal times. The x-axis represents Tohoku's earthquake month.

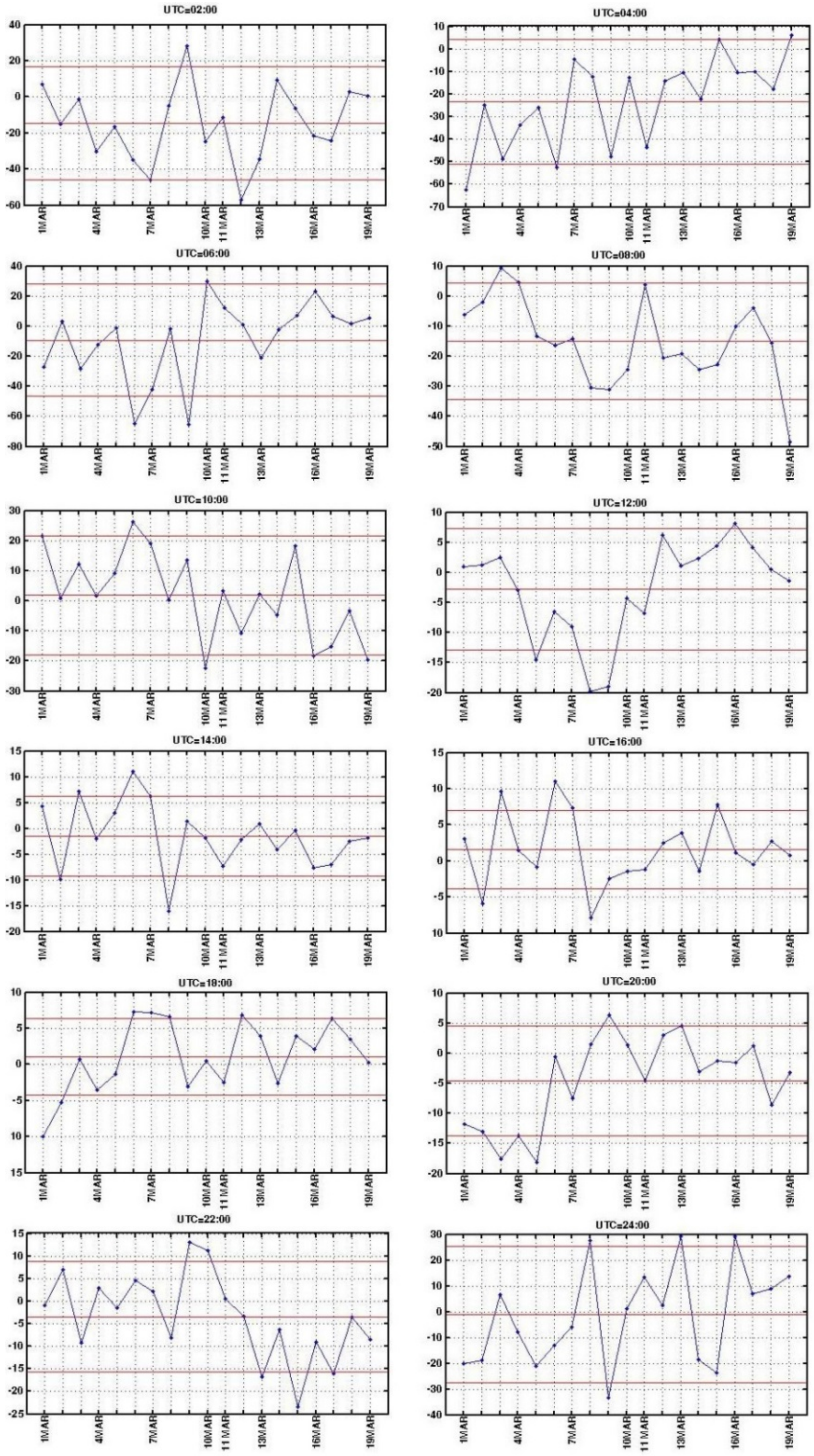

Figure 3. Variations of the differences between the observed and the predicted values of TEC on days selected as testing set at different universal times. The red horizontal lines indicate the upper and lower bounds (μ±kσ). The middle red horizontal line indicates the mean value (μ).The x-axis represents Tohoku's earthquake month.

Figure 4a reveals the differences between the observed and the predicted TEC values during the days selected as the test set. Figure 4b shows the variations of $DTEC$ (Eq. 9) from 1 February to 21 March in 2011 which is around Tohoku's earthquake date. Figure 4c indicates the detected TEC anomalies based on $|DTEC| > 2.5$. The noticeable anomalies are clearly seen from 4 days before the earthquake. Figure 4d demonstrates the detected TEC anomalies based on Eq. (10).

In order to show the efficiency of the proposed algorithm, the results obtained from this study have been compared with those from the mean, wavelet, Kalman filter and SVM methods. Table 2 demonstrates the details of the anomalies detected by these methods. As can be seen in Table 2, there exist good agreements between the proposed method and the other ones.

Conclusions

In the precursor studies, the anomaly detection is regarded as a crucial step. In this paper the SVT method has been used to detect anomalies. Results indicate that the difference between the observed and predicted TEC reaches the maximum value at the earthquake time during high geomagnetic activities. In this study, since the detected anomalies have been obtained in the quiet geomagnetic conditions by exerting the aforementioned constrains, they can be interpreted as the pre-earthquake ionospheric abnormal variations.

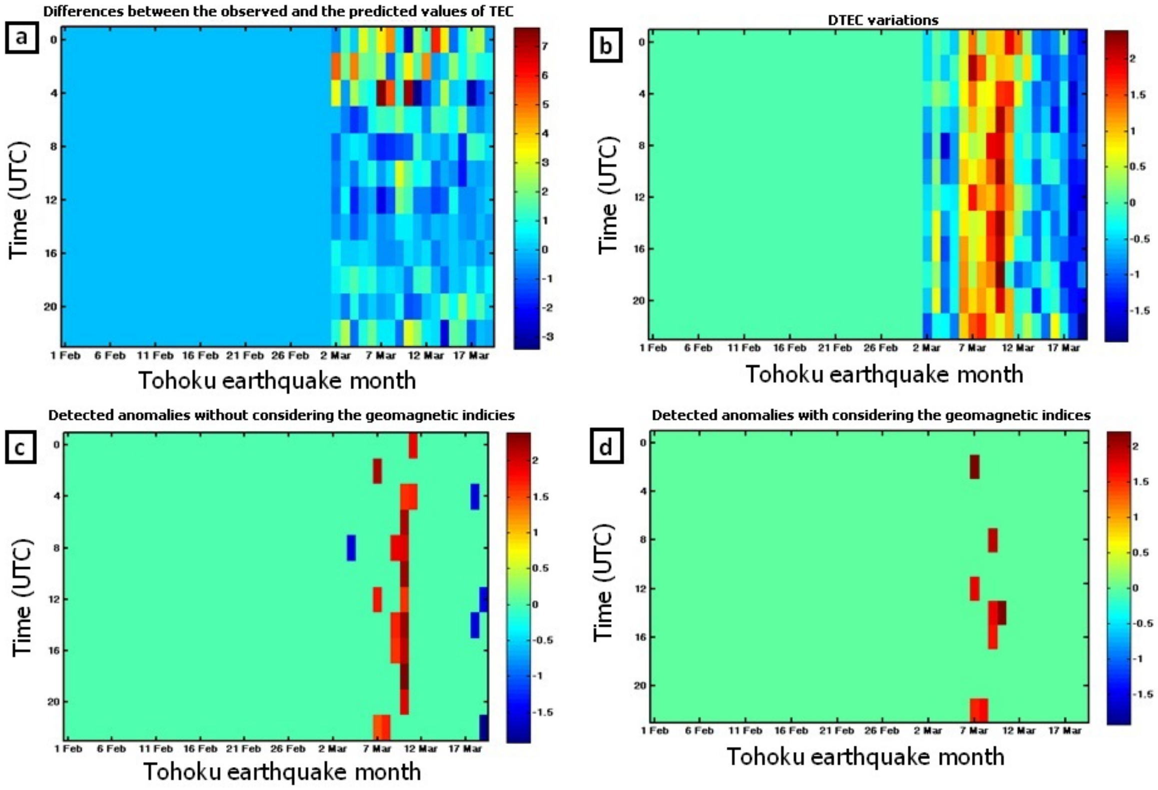

Figure 4. (a) Differences between the observed and the predicted values of TEC, (b) DTEC variations, (c) detected anomalies without considering the geomagnetic indices. (d) detected anomalies with considering the geomagnetic indices.

Table 2. Detected anomalies for the Tohoku earthquake (11 March 2011) using the median/interquartile, wavelet, Kalman filter and SVM methods. Day is relative to the earthquake day. Value calculated by $p = \pm 100(|Dx| - k) / k$ (Akhoondzadeh, 2012, 2013).

Mean			**Median**			**Wavelet**			**Kalman Filter**			**SVM**		
Day	**Time**	**Value**	**Day**	**Time**	**Value**	**Day**	**Time**	**Value**	**Day**	**Time**	**Value**	**Day**	**Time**	**Value**
2	14:00	5.43	3	14:00	67.30	1	12:00	16	3	14:00	15.75	6	03:00	10
3	12:00	7.26				1	12:00	12.8	3	16:00	112.33	Proposed method		
3	14:00	31.81				2	12:00	16	8	14:00	1.28	Day	Time	Value
						2	14:00	12.8				4	2	32
						3	12:00	9.2				4	12	21.6
						3	14:00	22				4	22	13.2
						3	16:00	1.6				3	22	18.3
						4	12:00	19.2				2	8	23.6
						4	14:00	22				2	14	16.8
						4	16:00	1.6				2	16	18.6
						5	12:00	4				1	14	52.17
						5	14:00	8.8						
						6	12:00	4						
						6	14:00	8.8						
						8	12:00	2						
						8	14:00	11.4						

Considering the results obtained, this algorithm can be utilized to predict the nonlinear time series. It is noteworthy that there are many scientists over the world working with the ionospheric data. Their objective is to characterize these anomalies and the kind of seismic events they are associated with, to learn how to automatically detect them in the data, to compare their occurrence with the seismic activity in order to understand their origin, and to define criteria which can be used in the future to make predictions. This is a long-term goal of our research, but the signal observed before the Mw 9 Tohoku earthquake will contribute to this task.

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