Iranian Journal of Geophysics, 2017, Page 71 - 84

A closer look at rock physics models and their assisted interpretation in seismic exploration

Mohammad Reza Saberi [1*]

[1] *CGG, Bordewijklaan 58, 2591 XR, The Hague, The Netherlands*

(Received: 19 February 2017, Accepted: 24 May 2017)

Abstract

Subsurface rocks and their fluid content along with their architecture affect reflected seismic waves through variations in their travel time, reflection amplitude, and phase within the field of exploration seismology. The combined effects of these factors make subsurface interpretation by using reflection waves very difficult. Therefore, assistance from other subsurface disciplines is needed if we intend to make a more accurate image of the subsurface. In this regard, rock physics acts as an integrated tool to combine subsurface information from different disciplines in a set of relationships between engineering (petrophysical) properties and their relevant geophysical variations, or more specifically, elastic variations. As a matter of fact, rock physics is required for a better understanding of rock properties if we intend to have a full understanding of our reservoir properties and their fluid content. This paper reviews some of the most important rock physics models and their application within the field of seismic exploration. These models are generally valid for the given conditions in which they are derived, and as a result, having a good understanding of their physical and geological limitations can help a lot with accurate rock physics modeling and interpretation. In this regard, this paper is an attempt to create a better understanding of such models, using different references and my personal experiences with these models. The application contexts of the models presented in this paper are not limited to the discussed scenarios. These scenarios are the ones that are commonly used and have shown a good prediction power in practice.

Keywords: rock physics, seismic velocities, elastic rock properties, rock properties, exploration seismology

[*]Corresponding author: reza.saberi@cgg.com

1 Introduction

Rock physics science has been rapidly evolving during the last few decades, from a theoretical science into a more practical approach (discipline) to address some of the most important subsurface problems. These developments in studying physical properties of minerals and fluids under different environmental conditions (like pressure and temperature) make it possible to find more accurate relationships for the purpose of interpreting and modelling desired scenarios within the field of geophysics, especially in seismic exploration and seismology. Between different disciplines in geophysical studies, seismic studies which work in the elastic domain are the most popular, and this is why most of rock physics studies and developments are focused on disciplines that work in elastic wave domains. This is also the main reason why rock physics models are sometimes referred to as petroelastic models (PEM) instead of rock physics models. The goal of these petroelastic model is to understand how lithology, porosity, confining stress and pore pressure, pore fluid type and saturation, anisotropy and degree of fracturing, temperature, and frequency influence the velocities and attenuation of compressional P- and S-waves in sedimentary rocks and vice versa (King, 2005). These relationships or models could have different sources, from laboratory experiments to theoretical principles, and combine concepts and principles from geology, geophysics, petrophysics, applied mathematics, and other disciplines (Sayer, 2013). There are lots of different rock physics relationships for various scenarios given in different sources (e.g. Voigt, 1890; Reuss, 1929; Gassmann, 1951; Hill, 1952; Biot, 1956; Hashin and Shtrikman, 1963; Berryman, 1980a; Berryman, 1980b; Hudson, 1980; Digby, 1981; Eberhart-Phillips, 1989; Krief et al., 1990; Greenberg and Castagna, 1992; Batzle and Wang, 1992; Brie et al., 1995; Ciz and Shapiro, 2007; Xu and Payne, 2009) which could potentially make modeling difficult without pre-knowledge of their assumptions and applicability.

Avseth *et al.* (2005) divided rock physics models into three general classes named as theoretical, empirical and heuristic models. This classification makes a clear distinction between the sources of different models and how they are derived. However, a closer look into general rock physics modeling workflows and procedures makes it clear that this classification needs to be modified for practical purposes. For a more practical application, rock physics models can be divided into (1) pore fluid models, (2) theoretical models, (3) empirical and heuristics models, and (4) hybrid models. This classification reflects the necessary steps needed in an actual rock physics modeling procedure. In this paper, I will describe each of these groups and will present some of the most important models in each category and their usual application for seismic exploration and seismic reservoir characterization purposes. Such models are used quite commonly, and can assist with interpretation of the observed subsurface elastic properties. The common practice in rock physics modeling is to model fluid and solid separately and then combine them together using relevant models. The fluid models which will be discussed first are commonly derived empirically under certain conditions in the laboratory. These relationships are then extrapolated to fit reservoir conditions. This is followed by modeling minerals and dry rocks, and finally by introducing modeled fluid inside the rock. The following contents are organized to reflect this procedure which

could, furthermore, help with building a rock from its constituents.

2 Rock physics models

In order for a better understanding of rock physics models, we need to classify them based on their derived origin which could be done into 4 different categories as (1) pore fluid models, (2) theoretical models, (3) empirical and heuristics models, and (4) hybrid models. In the following section, these groups will be discussed in more details.

2.1 Pore fluid models

Finding hydrocarbon (oil and/or gas) within a subsurface reservoir is one of the main goals for exploration seismology and seismic reservoir characterization. It is expected that seismic can be used to map pore fluid changes, along with their type and distribution, within a given reservoir. This elastic discrimination is based on different compressibility contrasts for pore fluids in the reservoir. It indicates that advanced knowledge about the compressibility of different pore fluids can help to delineate various pore fluid distributions. There are different empirical relationships such as Batzle and Wang (1992) in which elastic properties of the commonly encountered pore fluids in petroleum reservoirs are empirically estimated. These correlations which are widely used in the oil industry, calculate fluid elastic properties for an individual phase when environmental conditions (like temperature and pressure) or even fluid composition (like Gas-Oil-Ratio) are changed. In normal practice, you need to model each phase of the pore fluid within your reservoir (or along the well path in case of petrophysical interpretation) by changing pressure and temperature using one of the fluid models like Batzle and Wang (1992) equation. However, reservoir pores are normally filled with two or three fluids. This condition, which is also referred to as partial saturation, can happen either as homogenous or non-homogeneous (patchy) saturation which in turn depends on how the elastic waves are measured (ultrasonic, sonic or seismic measurements). The next step of your fluid modeling would be to mix different fluids together in order to make one effective fluid. Brie *et al.* (1995) suggested an empirical equation for mixing different fluids together. This equation is given by:

$$K_{mix} = \left(K_{brine} - K_{gas}\right) \cdot S_w^e + K_{gas} \,, \qquad (1)$$

where K refers to the bulk modulus and indices brine and gas relate bulk modulus to either of these fluids, and S_w is water saturation. The exponent e is an empirical constant ranging between 1 and 40. It should be calibrated to fit experimental data. When e = 1, Brie's formula is the same as the Voigt (1928) model, and the mixture shows the highest stiffness (patchy saturation). As e increases, the patchy saturation nears the characteristics of uniform saturation and nears a nearly uniform saturation at e = 40 which is equal to the Reuss (1929) model (which is also called Wood's equation). Gurevich and Lopatnikov (1995) and Pride *et al.* (2004) showed that patchy-saturation is probably the dominant fluid mixing scenario in reservoirs at low frequencies, and a normal fluid mixture is located between these two boundaries. Therefore, using Brie *et al.* (1995) with e = 3 to 6 is normally suggested for fluid mixing in conventional reservoirs.

2.2 Theoretical models

Theoretical models are primarily continuum mechanics approximations of the elastic or poroelastic properties of rocks. Elastic theories do not consider rocks as porous media but just as one elastic medium, however a real rock is a porous media consisting of a solid and

fluid. Therefore, poroelasticity, which is a continuum theory for the analysis of porous media (an elastic matrix containing interconnected fluid-saturated pores) is a better representative for describing subsurface rocks. The most frequently used model to describe poroelasticity processes in fluid-saturated porous materials is the Biot (1956) model. The Biot model is a high-frequency model which is equal to the Gassmann (1951) model for low frequencies. It predicts a frequency-dependent velocity in terms of dry rock properties, and accounts for the fluid flows caused by seismic wave propagation. When a seismic wave passes through a porous rock, two fluid flows are induced in the rock microstructure named as global (i.e. relative motion of the fluid and solid) and squirt flows (i.e. fluid movement from compliant cracks to stiff pores). The squirt flow which is more important in rocks with different pore stiffness is not included in the Biot model, and as a result, Biot model normally overpredicts velocities to some degree depending on the rock microstructure (Mavko, 1998). Therefore, different attempts have been made to include squirt flow into the Biot model like Biot–squirt (BISQ) model developed by Dvorkin and Nur (1993). This model accounts for both seismic wave induced fluid flows and attempted to modify Biot velocity for the squirt flow.

In general, theoretical models can be divided into (a) bound models, (b) inclusion based theories, (c) contact models, (d) transformations, and (e) computational models (Avseth et al., 2005). Below, each of these groups will be discussed briefly.

2.2.1 Boundary models

Boundary models define the range in which maximum and minimum elastic properties of a given rock can physically exist. Possible upper and lower elastic bounds can be predicted by knowing only the elastic moduli and their volume fractions for each constituent of the rock. The most common boundary models are Reuss (1929) and Voigt (1928) which model the lowest and highest possible bounds for mixing different minerals and fluids together. Reuss (1929) and Voigt (1928) models can be defined as:

$$M = \left[\sum_i v_i M_i^N\right]^{1/N}, \qquad (2)$$

where M_i and v_i are the elastic modulus and volume fraction for the i^{th} constituent of the rock. N = 1 and -1 will give the Voigt (1928) and Reuss (1929) models, respectively. Normally the average of these two models, which is named as Hill (1952), is suggested for mixing different minerals together. Then, the Hill (1952) model can be defined as:

$$M = \frac{\left[\sum_i v_i / M_i\right]^{-1} + \left[\sum_i v_i M_i\right]}{2}. \qquad (3)$$

Another commonly used boundary model is Hashin–Shtrikman (1963). This model assumes a different geometry compared with the Voigt and Reuss model and predicts a narrower bound between minimum and maximum elastic properties. Figure 1 compares these two bound models with each other and with their Hill (1952) average.

The range between the maximum and minimum velocity boundaries can also be reduced by applying the concept of critical porosity developed by Nur et al. (1995). Critical porosity is the porosity at which sediments are formed and all grains are in contact with each other. From the rock physics point of view, this porosity is the border between the solid load bearing (lower than critical porosity) and the fluid load bearing (higher than critical porosity) of a rock. In critical porosity models, critical porosity is assumed instead of the fluid point (ϕ=1), and this assumption makes the solution

area narrower (refer to Mavko et al., 1998). If we want to predict the effective elastic properties more specifically, where effects of various geometric details of the constituents are considered (grains and pores), we need more advanced models like inclusion-based theories.

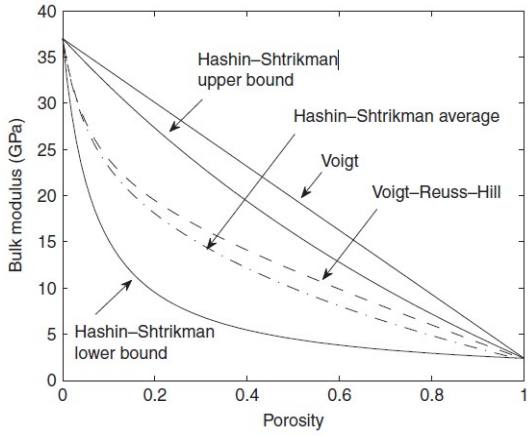

Figure 1. Comparison between boundary models and their Hill average. It shows that Hashin–Shtrikman (1963) boundaries are narrower than Voigt (1928) and Reuss (1929) bounds (Mavko et al., 1998).

2.2.2 Inclusion-based theories

Inclusion-based theories model wave velocity and attenuation based on scattering theory and approximates the rock as an elastic block of mineral perturbed by holes (porosity). They generally require the volume fraction of the constituents and physical and geometrical properties of the constituents, alone and relative to each other, for their solution. Various attempts have been made to account for the scattering effect of each inclusion. These solutions do not commonly depend on pressure and normal/tangential contact stiffness. They may consider the first order scattering term or the second and higher order scattering terms. The first order scattering solutions, such as Kuster and Toksöz (1974), does not account for pore to pore interactions. These interactions between pores are considered in the solutions with the second or higher order scattering terms such as differential equation medium (DEM) (Nishizawa, 1982), self-consistent approximation (SCA) (Berryman, 1980a,b) and T-matrix (Jakobsen et al., 2003a,b). Therefore, first order scattering models, which are represented by the Kuster and Toksöz (1974) solution ((K^*_{KT} and μ^*_{KT})) and are restricted to handling a dilute volume fraction of pores (lower porosity rocks), can be written as (Saberi, 2010):

$$\begin{cases} \left(K^*_{KT}-K_m\right)\dfrac{\left(K_m+\frac{4}{3}\mu_m\right)}{\left(K^*_{KT}+\frac{4}{3}\mu_m\right)}=\sum_{i=1}^{N}x_i\left(K_i-K_m\right)P^{mi} \\ \left(\mu^*_{KT}-\mu_m\right)\dfrac{\left(\mu_m+\zeta_m\right)}{\left(\mu^*_{KT}+\zeta_m\right)}=\sum_{i=1}^{N}x_i\left(\mu_i-\mu_m\right)Q^{mi} \\ \zeta=\dfrac{\mu\left(9K+8\mu\right)}{6\left(K+2\mu\right)} \end{cases} , (4)$$

where K and μ are bulk and shear modulus, and m and i refer to the background and inclusion materials. x_i is the volume fraction of inclusions summed over N different types of inclusion shapes. The coefficients P^{mi} and Q^{mi} describe the effect of an inclusion of material i in a background medium m and are given in Kuster and Toksöz (1974) and Mavko et al. (1998).

However, the second and higher orders scattering models allow for higher porosity rocks. The Differential Equation Medium (DEM) approach utilizes the principle of porosity growth to extend the results of the first order scattering solution (Kuster and Toksöz, 1974) to be valid at high porosities, while Self-Consistent Approximation (SCA) considers a uniform host material embedded with ellipsoidal inclusions (Berryman, 1980a,b). Both of these approaches simulate high-frequency saturated rock behavior and, therefore, are appropriate to apply to ultrasonic laboratory conditions. Furthermore, the visco-elastic effective medium theory of

Jakobsen et al. (2003a,b) (referred to as the T-matrix model) also takes into account global and local fluid flow, attenuation due to wave-induced fluid flow, anisotropy and various degrees of connectivity between pores. Therefore, it can handle modelling with complex situations (e.g. global and local fluid flow effects, different degrees of pore space connectivity, etc.) more accurately, but it has more unknown parameters to set and is more difficult to understand and calibrate. Lastly, Dræge (2006) combined the SCA and DEM in order to preserve connectivity between two mixing phases, and defined a new model named as CEMT (Combined Effective Medium Theory). CEMT first applies SCA for the volume concentrations between 40-60% and then DEM to preserve existing connectivity for each phase by changing their volume fractions.

The definition of pore shapes is perhaps one of the most challenging parameters in the inclusion-based models. Pore shapes are approximated by a parameter called as 'aspect ratio', which is defined as the ratio of short to long axis in an ellipsoid (assuming all pore shapes can be approximated ideally by an ellipsoid). There are various methods for determining aspect ratio which provide a better representation of the actual pore structure in reservoirs such as methods based on thin sections, or crossplotting techniques, or even some algorithms to match aspect ratio to the measured velocities. However, none of these methods can be considered as the best approach to determine pore aspect ratio, and each of them has its own cons and pros. The most important consideration in any of these methods is that of overfitting the calculated aspect ratio. Sometimes the calculated aspect ratio is treated like a fitting parameter, and is updated to get the least minimum error with the measured velocities, regardless of other parameters that may cause the actual observed errors (this is quite common in calculating aspect ratios using velocities).

2.2.3 Contact theories

These models typically consider rock to consist of packed spheres, and approximate it as a collection of separate grains. Their elastic properties are determined by deformability and stiffness of their grain-to-grain contact. The final solution is derived based on fundamental results for the deformation of two spheres in contact. Contact models, such as Hertz and Mindlin (Mindlin, 1949), Digby (1981) and Walton (1987), are more suitable for high porosity unconsolidated (shallow) sands where mechanical compaction is the predominant process during diagenesis. Within contact theory models, the Hertz and Mindlin model (Mindlin, 1949) is commonly used for modeling unconsolidated sandstones and is also sensitive to pressure changes (P_{eff}):

$$\begin{cases} K_{eff} = \left[\dfrac{C^2 (1-\varphi)^2 \mu_0^2}{18\pi^2 (1-\upsilon)^2} P_{eff} \right]^{1/3} \\ \mu_{eff} = \dfrac{5-4\upsilon}{5(2-\upsilon)} \left[\dfrac{3C^2 (1-\varphi)^2 \mu_0^2}{2\pi^2 (1-\upsilon)^2} P_{eff} \right]^{1/3} \end{cases}, \quad (5)$$

where C, ν, μ_0, υ, μ_0, ϕ and P_{eff} are coordination numbers (number of grain to grain contact point), Poisson's ratio, shear modulus of the grains, porosity and effective pressure, respectively.

This model assumes that there is no slip at the interface of two spheres under shear, therefore, it over-predicts for shear velocity. This means that shear velocity computed using the Hertz and Mindlin model needs to be adjusted by a coefficient named as the slip factor (Deng et al., 2006) ranging between 0 and 1. Furthermore, Dvorkin and Nur (1996) added mineral cement at contact grains (in order to include cement into sediments like modeling in the chemical

compaction zone) into the models using two schemes named contact cement (all cement deposited at grain contacts) and coating cement (cement deposited in a uniform layer around grains) models. These models depend on the amount of contact cement and on the properties of the cement as well as the grains. These theories help with identifying which type of cement prevails in sediment. For instance, quartz and clay cements can be distinguished if other parameters are known.

2.2.4 Transformation models

These models aim to transform the known elastic response of a given rock in one state to another state. The most common scenario for such models is the fluid substitution problem in which the velocities of rock for in situ saturation are known and you are interested in calculating the same rock velocity filled with another fluid. Gassmann's equation is the most important and widely used theory to solve the fluid substitution problem. This model is valid for homogenous rocks with connected pores at low frequencies. It is also assumed that there is no interaction between the fluid and solid and that hygroscopic fluid is part of the frame and is negligible. Gassmann predicts that shear modulus of the rock is not affected by fluid saturation, and bulk modulus of the saturated rock can be expressed as:

$$\begin{cases} \dfrac{K_{sat}}{K_0 - K_{sat}} = \dfrac{K_{dry}}{K_0 - K_{dry}} + \dfrac{K_{fl}}{\varphi\left(K_0 - K_{fl}\right)}, \\ \mu_{sat} = \mu_{dry} \end{cases} \quad (6)$$

where K_{sat}, K_{dry}, K_{fl} and K_0 are the bulk modulus for saturated rock, dry rock, fluid and mineral, respectively, and μ refers to the shear modulus of rock. Here, K_{fl} can be calculated using the Brie et al. (1995) equation if there are two or three fluids in the pores.

Although Gassmann has many assumptions, in practice it works well in many cases and is a very popular model for fluid substitution studies. Gassmann is relatively free of pore geometry assumptions and this is the main reason that its application in carbonate is questionable! It can overpredict, underpredict or even correctly predict velocities in carbonates. This means that using Gassmann in carbonates should be undertaken with care, and all other conditions should be taken into account. The most applicable scenario for Gassmann is in siliciclastics where it can be used with more confident. Another important point to mention is the fact that in the presence of anisotropy in rocks, anisotropic Gassmann is a more suitable choice for modeling the effect of fluid and can be beneficial for carbonates. Moreover, Brown and Korringa (1975) generalized Gassman's (1951) model for heterogeneous rocks, and furthermore, Ciz and Shapiro (2007) extended it for a solid that fills the pore space. Ciz and Shapiro (2007) model reads as:

$$\begin{cases} K_{sat}^{-1} = K_{dry}^{-1} - \dfrac{\left(K_{dry}^{-1} - K_0^{-1}\right)^2}{\varphi\left(K_{if}^{-1} - K_{\varphi}^{-1}\right) + \left(K_{dry}^{-1} - K_0^{-1}\right)}, \\ \mu_{sat}^{-1} = \mu_{dry}^{-1} - \dfrac{\left(\mu_{dry}^{-1} - \mu_0^{-1}\right)^2}{\varphi\left(\mu_{if}^{-1} - \mu_{\varphi}^{-1}\right) + \left(\mu_{dry}^{-1} - \mu_0^{-1}\right)} \end{cases} \quad (7)$$

where K_{ϕ} and K_{if} are the bulk modulus of pore space and pore filling solid, and ϕ is the porosity. The same indices on μ refer to the relevant shear modulus. This model predicts shear modulus changes with changing pore fluid material (unlike Gassmann), and is recommended for fluids that show shear modulus to some extent, such as bitumen in heavy oil reservoirs (tar sands).

2.2.5 Computational models

With recent development in computer science, it is now possible to reconstruct

a three-dimensional rock image with complex pore structure using digital imaging and even mathematically (like numerical methods) calculate some of its properties such as permeability. Here, more realistic three-dimensional image of the core can be constructed by using (a) Granular scale modeling, (b) 3D geostatistical reconstruction of 2D thin-section or SEM images, and (c) CT-scan of a small rock fragment (Richa 2010). The first method simulates grains in the reservoirs at deposition and then mathematically applies different diagenetic regimes to them and reconstructs the final rock, although it is not an exact replica of the real reservoir rock. The second approach uses grain-pore microgeometry from 2D thin-sections and uses geostatistical simulation to build a 3D digital binary rock. The last method represents the pore microstructure more accurately, as it employs a CT scan of a small fragment of the rock to build a 3D rock image. These techniques are becoming more and more popular in the rock physics world and with improving computer techniques, their application is also increasing.

2.3 Empirical and heuristic models

Empirical equations describe relationships between reservoir properties and their elastic responses through laboratory experiments. They generally assume some functional form and then determine coefficients by calibrating a regression to the data (Avseth et al., 2005). On the other hand, a heuristic model like the time average of Wyllie et al. (1956) defines P velocities only from the volume fractions of the various constituents and their velocities. Such a model emphasizes the relationship between various parameters in a certain way, through intuitive and non-rigorous means (Avseth et al., 2005). The time average model is a very simple model describing velocity for a mixture of solids and fluids as:

$$\frac{1}{V_p} = \frac{\varphi}{V_{fl}} + \frac{(1-\varphi)}{V_{p0}}, \tag{8}$$

where V_p, V_{fl}, V_{p0} and ϕ refer to saturated P-wave velocity, P-wave velocity for fluid, P-wave velocity for matrix and porosity, respectively.

The simplicity of this model makes it easy to understand and has some general applications such as quick velocity evaluation of a mixture of grains and fluids or even pore type discrimination. The inverse form of this model is sometimes used for porosity calculation which could inherently introduce some errors as it does not take into account many other effects on velocity such as pore type and clay effects. Anselmetti and Eberli (1999) used this equation (time average) in carbonates to discriminate between different pore types. They showed that the velocity relating to the inter-particle porosity lies around the time average, while the velocity for rocks with stiff and soft pores will lie above or below the time average, respectively. Raymer et al. (1980) improved the time average equation for consolidated rocks with low-to-medium porosity as below:

$$\begin{cases} V_p = (1-\varphi)^2 V_{p0} + \varphi V_{fl}, & \varphi < 37\% \\ \dfrac{1}{\rho V_p^2} = \dfrac{\varphi}{\rho_{fl} V_{fl}^2} + \dfrac{(1-\varphi)}{\rho_0 V_{p0}^2}, & \varphi > 47\% \end{cases}, \tag{9}$$

where ρ, ρ_{fl} and ρ_0 refer to the saturated rock density, fluid density and matrix density. In general, density can be calculated using the average of the rock constituents as:

$$\rho = \varphi\rho_0 + (1-\varphi)\rho_{fl}. \tag{10}$$

For intermediate porosities (between 37% and 47%) a simple interpolation between these two boundaries is

recommended. However, none of these equations determine the effects of clay minerals on velocity, and this is the main factor why these equations are normally accompanied by considerable errors when used for shaly sandstone. Eberhart-Phillips (1989) used a multivariate analysis to investigate the combined influences of effective pressure, porosity, and clay content for calculating velocities in water-saturated shaly sandstones. This relationship is given as:

$$\begin{cases} V_p = 5.77 - 6.94\varphi - 1.73\sqrt{V\_clay} \\ \qquad + 0.446(P_e - 1.0e^{16.7P_e}) \\ \\ V_s = 3.70 - 4.94\varphi - 1.57\sqrt{V\_clay} \\ \qquad + 0.361(P_e - 1.0e^{16.7P_e}) \end{cases}, \quad (11)$$

where V_clay refers to the clay content and P_e to the effective pressure. Effective pressure is the difference between overburden pressure and pore pressure, and should be used in Kbar units. Velocities (*Vp* and *Vs*) calculated using this model are pressure dependent and are given in km/s. This model gives an estimate of clay effects in sandstones along with effective pressure changes. Marion (1990) and Sams and Andera (2001) showed that clay can locate within sediments as structural, laminated, interstitial or dispersed. This defines whether the clay is load bearing or not and based on either of these scenarios, shaly sand velocity may change and therefore, determining clay structural distribution is very important for accurate velocity modeling in shaly sediments. Figure 2 shows the concept of these four different clay types.

Another important aspect for the success of any characterization project is the availability of both *Vp* and *Vs* at well locations. Most of the time either *Vs* data is not available or its quality is questionable. In these regard, having a relationship between *Vp* and *Vs* can help significantly for the cases where *Vs* data is missing. Castagna et al. (1985) presented an empirical relation named as the mudrock line that relates *Vp* to *Vs* velocities (both in Km/s) for brine-saturated clastic silicate rocks as:

$$V_p = 1.36 + 1.16V_s . \quad (12)$$

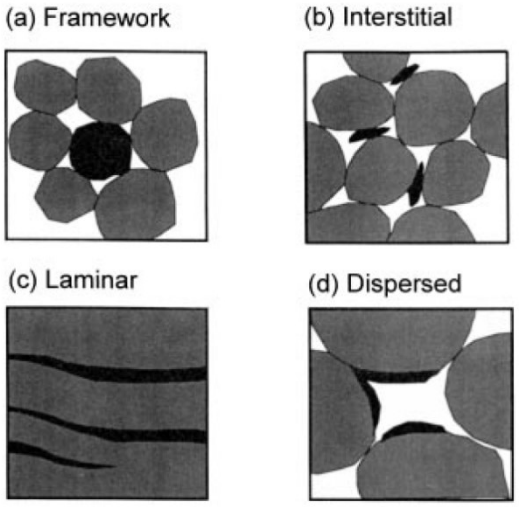

Figure 2 A schematic comparison between four different clay types that affect elastic properties differently (Sams and Andera, 2001).

This equation has wide applicability, especially in siliclastics and even in fluid discrimination in sandstones, using a parameter named as the fluid-factor introduced by Smith and Gildow (1987), based on P- and S-wave reflectivities. It assumes that any deviation from the mudrock line can be attributed to fluid changes using scaled differences between P- and S-wave reflectivities. This assumption makes use of the fluid factor equation mainly suitable for siliciclastics, as in those lithologies which do not follow the mudrock line deviations from it could potentially be interpreted as fluid changes. Furthermore, for *Vp* and *Vs* relationships, Krief et al. (1990) used the Raymer et al. (1980) dataset to find a relationship between Biot's coefficient and porosity, and then applied the

Gassmann (1951) equation to suggest a *Vp-Vs* prediction technique that very much resembles the critical porosity model (Mavko et al., 1998). This relationship is written as:

$$\frac{V_p^2 - V_{p_{fl}}^2}{V_s^2} = \frac{V_{p0}^2 - V_{p_{fl}}^2}{V_{s0}^2}, \qquad (13)$$

where *Vp* and *Vs* are the saturated P-wave and S-wave velocities. Indices 0 and *fl* refer to minerals and fluids.

Greenberg and Castagna (1992) combined the Gassmann equation and the Voigt-Reuss-Hill average to calculate *Vs* from *Vp* in multi-mineralic, brine-saturated rocks based on empirical, polynomial *Vp-Vs* relationships in pure mono-mineralic lithologies. The required input parameters to this model are *Vp*, lithology, saturation and porosity.

These three equations are quite commonly used to predict *Vs* from *Vp*, and depending on the situation either of them can be used to estimation S-wave velocity for different purposes.

2.4 Hybrid models

Hybrid models consist of a combination of two or more rock physics models in order to provide a better description of the reservoir. These models are widely used in the industry and are derived by connecting theoretical and/or empirical models together. There are various versions of the hybrid models which are adopted for different fields and scenarios, but the most important ones that are common in industry are: the Xu and White (1995) model, the Xu and Payne (2009) model, stiff and soft sand models (some of the empirical models like Krief et al. (1990) model can also be considered as hybrid model).

The Xu and White (1995) model connects three models together in order to calculate velocities of the saturated rock. It uses the time average model (Wyllie et al., 1956) to mix minerals together, and then the DEM equation is used to introduce dry pores into the effective minerals. Finally, the Gassmann equation is applied to introduce fluid into the dry pores. This model assumes two minerals (quartz and clay) with defined aspect ratios (clay aspect ratio much lower than quartz), and is more suitable for sandstone reservoirs. The Xu and Payne (2009) model, on the other hand, is designed for carbonate reservoirs. This model follows almost the same steps as the Xu and White (1995) model but assumes that total porosity consists of four pore types: (1) clay-related pores, (2) interparticle pores, (3) microcracks, and (4) stiff pores. This assumption makes it more suitable for sediments with varying pore structure such as carbonates (Figure 3).

The stiff and soft sand models are more concerned with granular media through combination of the Hertz and Mindlin model (Mindlin, 1949) with the modified version of Hashin–Shtrikman (1963) boundary model. They calculate elastic properties at critical porosity using the Hertz and Mindlin model and then extrapolate them for lower porosities using either modified upper or lower bounds of Hashin–Shtrikman model. The soft sand model is defined when the modified lower bound of Hashin–Shtrikman is used, and is suitable for uncemented sands where cements are deposited far away from grain contacts. Increasing the coordination number to 15 in this model gives an intermediate sand model. The stiff sand model uses the modified upper bound of Hashin–Shtrikman, and is suitable for cemented sands. It assumes that the initial porosity is reduced by deposition of cement at grain contacts.

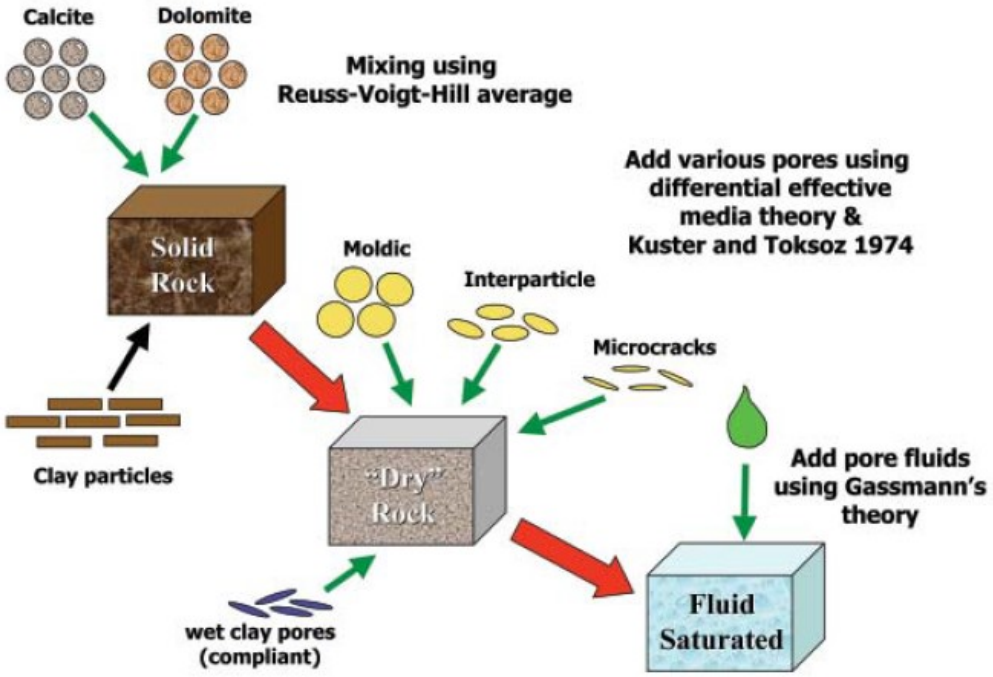

Figure 3. Carbonate rock physics modeling using Xu and Payne model (Xu and Payne, 2009).

A conceptual picture of these two models is shown in Figure 4. The modified version of the Hashin–Shtrikman (1963) boundary model includes the critical porosity concept in the Hashin–Shtrikman (1963) bounds in order to reduce the range between upper and lower boundaries.

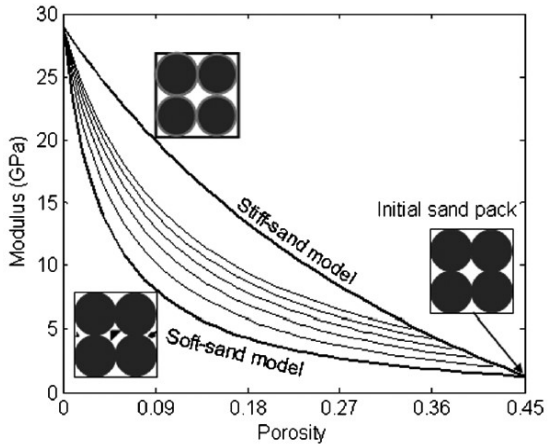

Figure 4. A conceptual picture showing stiff (modified Hashin–Shtrikman upper bound) and soft (modified Hashin–Shtrikman lower bound) sand models (Hossain et al., 2011).

3 Discussion

Although substantial effort has been made to develop new models for more common scenarios in subsurface but geology still is so complex that it cannot be explained by such simple models. These models are normally aimed at predicting a correct trend for your geology, and if you intend to make a more detailed calculation, more data needs to be input into your rock physics model. Moreover, anisotropy, dispersion, attenuation and difference in measurement scales can influence your measured data, and this is a big challenge during rock physics modeling. These effects should be accounted for at different steps of your modeling procedure in order to have a better prediction of elastics. In this regard the Backus (1962) and Hudson (1980) models are respectively the common models used for upscaling and crack induced anisotropy modeling. Backus (1962) showed that at the long wavelength limit, a stratified medium

composed of isotropic layers will behave like a transverse isotropic (TI) medium, and derived the effective elastic constants for such a medium. Saberi (2016) used forward and inverse method on Backus models in order to model anisotropy in a vertical isotropic medium (VTI) using a conventional log suit. Hudson (1980) derived the effective elastic constants of a cracked medium by modeling an isotropic background superimposed with oriented cracks. This model is quite commonly used in carbonates for modelling fracture effects on velocities and also modeling azimuthal inversion studies for fracture characterization.

The inputs to rock physics models are normally reservoir properties (e.g. porosity, saturation, etc.) and the elastic properties (e.g. bulk and shear modulus, P-wave velocity, etc.) are calculated accordingly. This approach, which is also referred to as rock physics forward modeling, is quite common for modeling velocities for petrophysical logs or even scenario modeling. The reverse procedure which is referred to as inverse rock physics modeling is becoming more important in the seismic reservoir characterization. In this approach, reservoir properties are determined from elastic properties, which are normally derived from seismic velocities or inversion methods. Using geostatistical techniques during rock physics modeling, or even interpreting rock physics results, will provide an estimate of error and uncertainty analysis. Statistical rock physics modeling is a very useful approach when you are not confident of the inputs into your model, or even the model itself.

4 Conclusions

Rock physics analysis and modeling have now become an important step in any reservoir characterization project. This means that knowledge about the rock properties of the reservoir is needed if we want to have a better understanding of our reservoir. This paper reviews some of the most important rock physics models and their applications on different lithologies. These models are currently the most commonly used, although there are many other models that may have better results than the ones introduced in this paper. This paper is just aimed at summarizing the most common models and collecting them in one place for a better comparison, and also to highligh their advantages and disadvantages.

References

Anselmetti, F. S. and Eberli, G. P., 1999, The velocity deviation log: A tool to predict pore type and permeability trends in carbonate drill holes from sonic and porosity or density logs: American Association of Petroleum Geologist, **83**, 450–466.

Avseth, P., Mukerji, T. and Mavko, G., 2005, Quantitative seismic interpretation: Applying Rock Physics Tool to Reduce Interpretation Risk (First Edition): Cambridge University Press, Cambridge, UK.

Backus, G. E., 1962, Long-wave elastic anisotropy produced by horizontal layering: Journal of Geophysical Research, **67**, 4427–4440.

Batzle, M. and Wang, Z., 1992, Seismic properties of pore fluids: Geophysics, **57**, 1396–1408.

Brie, A., Pampuri, F., Marsala, A. F., and Meazza, O., 1995, Shear sonic interpretation in gas-bearing sands: SPE 30595, 701–710.

Berryman, J. G., 1980a, Long-wavelength propagation in composite elastic media I. Spherical inclusions: Journal of Acoustic Society of America, **68**, 1809–1819.

Berryman, J. G., 1980b, Long-wavelength propagation in composite elastic media II. Ellipsoidal inclusions: Journal of Acoustic Society of America, **68**, 1820–1831.

Biot, M. A., 1956, Theory of propagation of elastic waves in a fluid saturated porous solid. I. Low frequency range and II. Higher frequency range: Journal of Acoustical Society of America, **28**, 168–191.

Brown, R. and Korringa, J., 1975, On the dependence of the elastic properties of a porous rock on the compressibility of the pore fluid: Geophysics, **40**, 608–616.

Castagna, J. P., Batzle, M. L. and Eastwood, R. L., 1985, Relationships between

compressional wave and shear wave velocities in clastic silicate rocks: Geophysics, **50**, 571–581.

Ciz, R. and Shapiro, S., 2007, Generalization of Gassmann equations for porous media saturated with a solid material: Geophysics, **72**, A75–A79.

Deng, J. X., Han, D. and Liu, J., 2006, The effects of geologic parameter variation on the A-B Cross-plot of sand reservoir: Fluid/DHI Annual Meeting.

Digby, P. J., 1981, The effective elastic moduli of porous granular rocks: Journal of Applied Mechanics, **48**, 803–808.

Dræge, A., 2006, Impact of Diagenesis on Seismic Properties of Siliciclastic Rocks: Ph. D. dissertation, University of Bergen, Norway.

Dvorkin, J. and Nur, A., 1993, Dynamic poroelasticity: a unified model with the squirt and the Biot mechanisms: Geophysics, **58**, 524–533.

Dvorkin, J. and Nur, A., 1996, Elasticity of high-porosity sandstones, Theory for two North Sea data sets: Geophysics, **61**, 559–564.

Eberhart-Phillips, D. M., 1989, Investigation of Crustal Structure and Active Tectonic Processes in the Coast Ranges, Central California: Ph. D. dissertation, Stanford University, USA.

Gassmann, F., 1951, Uber die Elastizitat poroser Medien: Vier. der Natur. Gesellschaft Zurich, **96**, 1–23.

Greenberg, M. L. and Castagna, J. P., 1992, Shear-wave velocity estimation in porous rocks: Theoretical formulation, preliminary verification and applications: Geophysical Prospecting, **40**, 195–209.

Gurevich, B. and Lopatnikov S. L., 1995, Velocity and attenuation of elastic waves in finely layered porous rocks: Geophysical Journal International, **121**, 933–947.

Hashin, Z. and Shtrikman, S., 1963, A variational approach to the elastic behavior of multiphase materials: Journal of Mechanics and Physics of Solids, **11**, 127–140.

Hill, R., 1952, The elastic behavior of crystalline aggregate: Proceeding of Physical Society, **65**, 349–354.

Hudson, J. A., 1980, Overall properties of a cracked solid: Mathematical Proceedings of the Cambridge Philosophical Society, **88**, 371–384.

Hossain, Z., Mukerji, T., Dvorkin, J. and Fabricius, I. L., 2011, Rock physics model of glauconitic greensand from the North Sea: Geophysics, **76**, E199-E209.

Jakobsen, M., Hudson, J. A., and Johansen, T. A., 2003a, T-matrix approach to shale acoustics: Geophysical Journal International, **154**, 533–558.

Jakobsen, M., Johansen, T. A., and McCann, C., 2003b, The acoustic signature of fluid flow in complex porous media: Journal of Applied Geophysics, **54**, 219–246.

King, M. S., 2005, Rock-physics developments in seismic exploration: A personal 50-year perspective: Geophysics, **70**, 3ND–8ND.

Krief, M., Garat, J., Stellingwerff, J., and Ventre, J., 1990, A petrophysical interpretation using the velocities of P and S waves (full-waveform sonic): Log Analyst, **31**, 355–369.

Kuster, G. T., and Toksöz, M. N., 1974, Velocity and attenuation of seismic waves in two phase media: Part I. Theoretical formulations: Geophysics, **39**, 587–606.

Marion, D., 1990, Acoustical, Mechanical and Transport Properties of Sediments and Granular Materials: Ph. D. dissertation, Stanford University.

Mavko, G., Mukerji, T. and Dvorkin, J., 1998, The rock physics handbook: Cambridge University Press, Cambridge, UK.

Mindlin, R. D., 1949, Compliance of elastic bodies in contact: Journal of Applied Mechanics, **16**, 259–268.

Nishizawa, O., 1982, Seismic velocity anisotropy in a medium containing oriented cracks transversely isotropic case: Journal of Physics of the Earth, **30**, 331–347.

Nur, A., Mavko, G., Dvorkin, J. and Gal, D., 1995, Critical porosity: the key to relating physical properties to porosity in rocks: In Proceeding of 65th Annual International Meeting, Society Exploration Geophysicist, 878.

Pride, S. R., Berryman J. G. and Harris J. M., 2004, Seismic attenuation due to wave-induced flow: Journal of Geophysical Research, **109**, B01201.

Raymer, L. L., Hunt, E. R., and Gardner, J. S., 1980, An improved sonic transit time-to-porosity transform: Transcript for Society of Professional Well Log Analysts, 21st Annual Logging Symposium, Paper P.

Reuss, A., 1929, Berechnung der Fliessgrenzen vonMischkristallen aufGrund der Plastizita¨tsbedingung fu¨r Einkristalle: Z. Ang. Math. Mech., **9**, 49–58.

Richa R., 2010, Preservation of Transport Properties Trends: Computational Rock Physics Approach: Ph. D. dissertation, Stanford University, USA.

Saberi, M. R., 2010: An Integrated Approach for Seismic Characterization of Carbonates, Ph. D. dissertation, University of Bergen, Norway.

Saberi, M. R., 2016, Modeling an elastic stiffness tensor in a transverse isotropic subsurface medium: International application Patent No: WO 2016/083893 A1.

Sams M. S. and Andera M. A., 2001, The effect of clay distribution on the elastic properties of sandstones: Geophysical Prospecting, **49**, 128–150.

Sayer, C., 2013, Introduction: Rock Physics for Reservoir Exploration, Characterisation and Monitoring: Geophysical Prospecting, **61**, 251–253.

Smith, G. C. and Gidlow, P. M., 1987, Weighted stacking for rock property estimation and detection of gas: Geophysical Prospecting, **35**, 993–1014.

Thomas, E. C. and Stieber, S. J., 1975, The distribution of shale in sandstones and its effect upon porosity: In Transcripts of 16th Annual Logging Symposium of the SPWLA, paper T.

Voigt, W., 1890, Bestimmung der Elastizita¨tskonstanten des brasilianischen Turmalines: Annual Review of Physical Chemistry, **41**, 712–729.

Walton, K., 1987, The effective elastic moduli of a random packing of spheres: Journal of Mechanics and Physics of Solids, **35**, 213–226.

Wang, Z., 2001, Fundamentals of seismic rock physics: Geophysics, **66**, 398–412.

Xu, S. and White, R. E., 1995, A new velocity model for clay-sand mixtures: Geophysical Prospecting, **43**, 91–118.

Xu, S. and Payne, M. A., 2009, Modeling elastic properties in carbonate rocks: The Leading Edge, **28**, 66–74.