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# A Review of Constitutive Modeling of Unsaturated Soils

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## Abstract

The mechanics of unsaturated soils are integral to understanding and predicting the behavior of diverse geotechnical and geo-environmental systems, including natural slopes, engineered embankments, landfill covers, and agricultural fields. Unlike saturated soils, unsaturated soils exhibit a complex interplay among air, water, and solid phases, in which suction and partial saturation are pivotal in governing stress-strain responses, volume changes, and fluid flow processes. Over the past several decades, extensive theoretical, experimental, and computational efforts have culminated in sophisticated constitutive models, reflecting the need to include moisture content and suction as additional state variables.This article comprehensively reviews these modeling endeavors, tracing historical developments from empirical extensions of saturated soil models to contemporary elasto-plastic, multi-scale, and data-driven frameworks. Emphasis is placed on the evolution of stress state variables, the role of hydraulic hysteresis, and the bidirectional coupling between mechanical and water retention behavior. Special attention is given to recent models that incorporate bound water structure and dehydration mechanisms in expansive clays, highlighting their influence on retention properties, suction, and thermomechanical responses. The paper also explores advances in small-strain stiffness modeling (which is crucial for predicting seismic behavior of unsaturated soils), machine learning integration, and coupled thermo-hydro-mechanical-chemical (THMC) processes. Practical challenges, including parameter calibration, are examined. The paper also offers examples of prominent constitutive models, detailing their mathematical formulations and underlying assumptions. Current trends, including the integration of machine learning, are evaluated, and future research directions are proposed, underscoring the importance of interdisciplinary collaborations and long-term monitoring to refine and validate constitutive models for unsaturated soils.

**Keywords:** Unsaturated soils, small strain shear stiffness, constitutive modelling, thermo-hydro-mechanical model, water retention curve

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## 1 Introduction

The behavior of unsaturated soils, where water and air occupy the pore spaces, is essential to understanding geotechnical and environmental systems. These soils exhibit complex interactions between solid, liquid, and gas phases, with suction and partial saturation critical in influencing the soil's stress-strain responses, volume changes, and fluid flow behaviors. Applications ranging from slope stability to agriculture require accurate models of these behaviors to predict soil responses to varying environmental conditions.

For decades, advancements in theoretical, experimental, and computational research have led to development of sophisticated constitutive models. Unlike saturated soils, unsaturated soils require additional state variables like moisture content and suction. This paper reviews the evolution of these models, focusing on their theoretical foundations, key developments, and challenges in practice. It highlights the notable models that have shaped the field while addressing current trends, such as machine learning integration. In conclusion, the paper proposes future research directions to refine and validate these models, emphasizing the importance of interdisciplinary collaboration.

## 2 Historical Overview and Key Developments

The study of unsaturated soils has progressed significantly from its origins in hydraulic studies, primarily focused on water flow and infiltration. Early attempts to extend saturated soil mechanics to unsaturated soils introduced an empirical suction term in failure envelopes, notably by Aitchison (1965) and Escario and Saez (1973), who proposed the concept of "apparent cohesion" based on suction. Although useful in certain contexts, these models were limited in capturing key behaviors such as wetting-induced collapse.

A major shift occurred in the 1970s with the introduction of two independent stress variables—net normal stress and suction by Fredlund and Morgenstern (1977). This development laid the groundwork for more robust models, particularly those based on critical state soil mechanics (Roscoe and Burland (1968)). In this period, models began incorporating suction and stress into frameworks based on plasticity theory. Bishop (1959) effective stress approach, which included a parameter ($\chi$) to account for suction, provided a key theoretical advancement.

By the 1990s, elastoplastic constitutive models, such as the Barcelona Basic Model (BBM) by Alonso et al. (1990), became pivotal in characterizing unsaturated soils. These models introduced independent stress variables and demonstrated the ability to model collapse and other complex behaviors under unsaturated conditions. Over time, models expanded to address coupled processes (heat, chemical interactions) and multi-scale behaviors, providing a more holistic understanding of unsaturated soils.

More recently, machine learning and data-driven approaches have revolutionized model development, allowing for more precise calibration and validation. Coupled models integrating mechanical, hydraulic, and thermal behaviors have become increasingly important in predicting soil behavior under real-world conditions. Published constitutive formulations for unsaturated soils dated 1990 – 2024 were reviewed. Forty peer-reviewed models with a complete mathematical description were retained; their chronological list appears in Table 1.

Each model was assigned five independent categories to compare formulations founded on different assumptions within a standard frame. The first category records the stress-variable set, net normal stress, matric suction, Bishop effective stress, skeleton stress, or a stress–saturation pair, because that choice governs work conjugacy and the definition of effective stress.

The second category identifies the constitutive framework, distinguishing critical-state elasto-plasticity, bounding-surface plasticity, hypoplasticity, incremental plasticity, and recent thermo or chemo-extended variants, including large-strain forms. The third category states the treatment of hysteresis, which may be absent, limited to hydraulic effects through scanning water-retention curves, or extended to mechanical cycles utilizing kinematic or bounding-surface mechanisms; specialised thermal or gas flow hysteresis is also noted. The fourth category records the adopted dilatancy or flow rule, critical state line-based, state parameter dependent, empirical, hypoplastic, or small-strain specific. The fifth category specifies the coupling capability, distinguishing (i) mechanical formulations that require prescribed suction or degree of saturation, (ii) fully coupled hydro-mechanical models that solve the retention relation together with the mechanical response, and (iii) further extensions that incorporate temperature, chemistry, gas pressure or large strain kinematics.

The published equations were examined for every model to extract the chosen stress variables, internal state variables, yield and hardening laws, hysteresis representation, dilatancy formulation, additional coupling fields, validation paths, and numerical integration strategy. These descriptors provide the basis for the comparative discussion in Sections 3 to 7. Section 3 recalls the saturated antecedents of the unsaturated models; Section 4 evaluates the stress-variable choices; Section 5 contrasts the hysteresis strategies; Section 6 analyses the various dilatancy formulations and their cyclic performance; and Section 7 reviews recent thermo, chemo, and large-strain extensions.

**Table 1.** Constitutive models for unsaturated soils.

| # | Model | Stress variable | Constitutive framework | Hysteresis treatment | Dilatancy/flow rule | Coupling level |
|---|---|---|---|---|---|---|
| 1 | Alonso et al. (1990) | Net normal stress and matric suction | Critical-state elasto-plastic | Hydraulic hysteresis via scanning curves | CSL flow rule | Mechanical-only model – analyst must prescribe matric suction; the model does not compute degree of saturation. |
| 2 | Gens and Alonso (1992) | Net normal stress and matric suction | CS elasto-plastic (expansive) | Hydraulic and mechanical swell–shrink hysteresis | CSL shift | Mechanical-only model – suction prescribed; hydraulic hysteresis modifies mechanical response. |
| 3 | Wheeler and Sivakumar (1995) | Bishop effective stress | Critical-state elasto-plastic | None | CSL flow rule | Mechanical-only model – degree of saturation prescribed; no hydraulic prediction. |
| 4 | Alonso et al. (1999) | Net normal stress and matric suction | CS elasto-plastic (double structure) | Hydraulic hysteresis on swelling loops | CSL shift | Mechanical-only model – suction prescribed; hydraulic hysteresis modifies mechanical response. |
| 5 | Oldecop and Alonso (2001) | Net normal stress | Incremental plasticity (rockfill) | None | Empirical compressibility | Mechanical-only model – dry material; no hydraulic variables involved. |
| 6 | Loret and Khalili (2002) | Bishop effective stress | Elastic-plastic (energy) | None | Yield only | Mechanical-only model – degree of |

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| | | | | | | saturation prescribed; no hydraulic prediction. |
| 7 | Wheeler et al. (2003) | Bishop effective stress | Critical-state elasto-plastic | Hydraulic and mechanical hysteresis via kinematic hardening | CSL flow rule | Coupled hydro-mechanical model – predicts degree of saturation and mechanical response with hydraulic and mechanical hysteresis. |
| 8 | Gallipoli et al. (2003) | Bishop effective stress | Critical-state elasto-plastic | Hydraulic hysteresis via scanning curves | CSL flow rule | Coupled hydro-mechanical model – predicts saturation and mechanical behaviour with hydraulic hysteresis. |
| 9 | Chiu and Ng (2003) | Bishop effective stress | State-dependent CS el-pl | None | State-parameter dilatancy | Mechanical-only model – degree of saturation prescribed; no hydraulic prediction. |
| 10 | Tamagnini (2004) | Net normal stress and matric suction | Extended Cam-Clay | Hydraulic hysteresis via scanning curves | CSL flow rule | Mechanical-only model – suction prescribed; hydraulic hysteresis modifies stiffness. |
| 11 | Sheng et al. (2004) | Net normal stress and matric suction | Thermo-elasto-plastic critical-state | Hydraulic hysteresis via scanning curves | CSL flow rule | Coupled thermo–hydro–mechanical model – predicts temperature, suction and mechanical response with hydraulic hysteresis. |
| 12 | Fredlund and Pham (2006) | Net normal stress and matric suction | Volume–mass el-pl | None | Empirical | Mechanical-only model – suction prescribed; no hydraulic prediction. |
| 13 | Sun et al. (2007) | Net normal stress and matric suction | Density-dependent elastic-plastic | Hydraulic hysteresis via scanning curves | CSL shift by density | Mechanical-only model – suction prescribed; hydraulic hysteresis modifies mechanical response. |
| 14 | Li (2007) | Bishop effective stress | Thermodynamics-based el-pl | None | CSL flow rule | Mechanical-only model – degree of saturation prescribed; no hydraulic prediction. |
| 15 | Gallipoli et al. (2008) | Bishop effective stress | Critical-state elasto-plastic | Hydraulic hysteresis via scanning curves | CSL at critical state | Coupled hydro-mechanical model – predicts saturation and mechanical behaviour with hydraulic hysteresis. |
| 16 | Khalili et al. (2008) | Bishop effective stress + suction-rate term | Coupled elastic-plastic | Hydraulic and mechanical cyclic hysteresis | Empirical | Hydro-mechanical cyclic model – computes cyclic interaction between suction-rate |

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| | | | | | | changes and mechanical response. |
| 17 | Sheng et al. (2008) | Net normal stress and matric suction | El-pl (independent vars.) | None | CSL flow rule | Mechanical-only model – suction prescribed; no hydraulic prediction. |
| 18 | Stropeit et al. (2008) | Bishop effective stress | Anisotropic elastic-plastic | None | CSL flow rule | Mechanical-only model – degree of saturation prescribed; no hydraulic prediction. |
| 19 | Zhang and Ikariya (2011) | Skeleton stress and saturation | Elastic-plastic skeleton model | None | CSL flow rule | Mechanical-only model – degree of saturation prescribed; no hydraulic prediction. |
| 20 | Zhou et al, (2012a, b) | Net normal stress and saturation | Elastic-plastic in stress–saturation space | Hydraulic hysteresis via scanning curves | CSL shift | Coupled hydro-mechanical model – predicts saturation and mechanical behaviour with hydraulic hysteresis. |
| 21 | Liu and Muraleetharan (2012) | Net stress and suction | Coupled elastic-plastic | Hydraulic hysteresis via scanning curves | CSL flow rule | Coupled hydro-mechanical model – predicts saturation and mechanical behaviour with hydraulic hysteresis. |
| 22 | Lloret-Cabot et al. (2013) | Bishop effective stress | 3-D el-pl with retention coupling | Hydraulic hysteresis via scanning curves | CSL shift | Coupled hydro-mechanical model – predicts saturation and mechanical behaviour with hydraulic hysteresis. |
| 23 | Ghasemzadeh and Amiri (2013) | Net stress and suction | Isotropic elastic-plastic | None | CSL flow rule | Mechanical-only model – suction prescribed; no hydraulic prediction. |
| 24 | Wong and Mašín (2014) | Bishop effective stress | Hypoplasticity | Hydraulic hysteresis implicitly modifies stiffness | Hypoplastic flow rule | Mechanical-only model – degree of saturation prescribed; hydraulic hysteresis affects stiffness. |
| 25 | Zhou and Sheng (2015) | Net stress and saturation | Advanced elastic-plastic | Hydraulic hysteresis via scanning curves | State-parameter dilatancy | Coupled hydro-mechanical model – predicts saturation and mechanical response with hydraulic hysteresis. |
| 26 | Zhou et al. (2015) | Bishop effective stress | Bounding-surface plasticity | None | Empirical small-strain dilatancy | Mechanical-only model – degree of saturation prescribed; no hydraulic prediction. |
| 27 | Tourchi and Hamidi (2015) | Net stress, suction and temperature | CS thermo-elasto-plastic | None | CSL thermo-dilatancy | Coupled thermo–hydro–mechanical model – predicts temperature, suction and mechanical response. |

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| 28 | Lloret-Cabot et al. (2017) | Net stress–suction or Bishop stress | Unified mech.–retention | Hydraulic hysteresis via scanning curves | CSL flow rule | Coupled hydro-mechanical model – predicts saturation and mechanical response (user switchable). |
| 29 | Chong (2017) | Net stress and suction | Dynamic non-linear | Mechanical hysteresis (Masing) | Empirical cyclic dilatancy | Mechanical-only model – suction prescribed; mechanical hysteresis captured, no hydraulic prediction. |
| 30 | Gholizadeh and Latifi (2018) | Bishop effective stress | Coupled elastic-plastic | Hydraulic hysteresis via scanning curves | CSL flow rule | Coupled hydro-mechanical model – predicts saturation and mechanical behaviour with hydraulic hysteresis. |
| 31 | Li and Yang (2018) | Net stress and suction | OCR-dependent elastic-plastic | Hydraulic hysteresis via scanning curves | CSL shift by OCR | Mechanical-only model – suction prescribed; hydraulic hysteresis affects response. |
| 32 | Bruno and Gallipoli (2019) | Bishop effective stress | Bounding-surface plasticity | Hydraulic and mechanical hysteresis via bounding surface | CSL flow rule | Coupled hydro-mechanical model – predicts saturation and mechanical behaviour with hydraulic and mechanical hysteresis. |
| 33 | Cheng et al. (2020) | Net stress, suction and temperature | Two-surface thermo-elasto-plastic | Thermal hysteresis in heat–cool cycles | CSL shift by temperature | Coupled thermo–hydro–mechanical model – predicts temperature, suction and mechanical response with thermal hysteresis. |
| 34 | Xiong (2020) | Bishop stress (finite strain) | Large-strain elastic-plastic | Hydraulic hysteresis via scanning curves | CSL flow rule | Large-strain mechanical-only model – analyst must prescribe suction; model handles finite deformation but not hydraulic variables. |
| 35 | Mahmoodabadi and Bryson (2021) | Net stress and suction | Fully coupled elastic-plastic | Hydraulic hysteresis via scanning curves | CSL flow rule | Coupled hydro-mechanical model – predicts saturation and mechanical behaviour with hydraulic hysteresis. |
| 36 | Moghaddasi et al. (2021) | Bishop effective stress and matric suction | Bounding-surface plasticity with bonding degradation | Hydraulic hysteresis via scanning SWRC; mechanical via bounding surface | CSL-based with bonding factor | Mechanical-only – suction prescribed |
| 37 | Tafili and Machaček (2023) | Bishop effective stress | Hypoplasticity | Hydraulic and mechanical hysteresis | Hypoplastic flow rule | Mechanical-only model – suction |

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| | | | | via internal variables | | prescribed; hydraulic and mechanical hysteresis affect response. |
| 38 | Lu et al. (2023) | Bishop stress and chemical concentration | Chemo-thermo-hydro-mechanical plasticity | Hydraulic hysteresis via scanning curves | CSL shift by chemistry | Coupled chemo–thermo-hydro-mechanical model – predicts chemistry, temperature, suction and mechanical behaviour. |
| 39 | Corman (2023) | Bishop stress and gas pressure | Multi-scale elastic-plastic (gas migration) | Gas-flow hysteresis through pressure-dependent retention | Empirical | Coupled thermo–hydro-mechanical model – predicts gas pressure, suction and mechanical response. |
| 40 | Sojoudi and Li (2023) | Net stress and temperature | Thermo-elastic-plastic | None | CSL flow rule | Coupled thermo–hydro-mechanical model – predicts temperature, suction and mechanical behaviour. |
| 41 | Quevedo et al. (2024) | Net stress and suction | Elastic-plastic with compaction | Hydraulic hysteresis via scanning curves | CSL flow rule | Coupled hydro-mechanical model – predicts saturation and mechanical behaviour with hydraulic hysteresis. |
| 42 | Yang et al. (2024) | Net normal stress and matric suction | Hypoplastic with suction-dependent structural factor | Hydraulic hysteresis via structural collapse; mechanical implicit | Implicit hypoplastic dilatancy | Mechanical-only – suction prescribed |
| 43 | Kadivar et al. (2024) | Bishop effective stress and matric suction | Hyper-elastic bounding-surface plasticity | Hydraulic hysteresis via scanning SWRC; mechanical via bounding surface | CSL-based within bounding surface | Mechanical-only – suction prescribed |

## 3 Saturated roots of unsaturated constitutive models

A large proportion of the formulations gathered in Table 1 extend a pre-existing model for saturated soils. Identifying that saturated "parent" clarifies both the mathematical structure inherited and the additional mechanisms introduced for partial saturation as described in Table 2.

**Table 2.** Saturated antecedent frameworks of unsaturated models.

| # | Unsaturated constitutive model | Saturated parent framework retained | Key saturated reference |
|---|---|---|---|
| 1 | Alonso et al. (1990) | Modified Cam-Clay | Roscoe and Burland (1968) |
| 2 | Gens and Alonso (1992) | Modified Cam-Clay | Roscoe and Burland (1968) |
| 3 | Wheeler and Sivakumar (1995) | Modified Cam-Clay | Roscoe and Burland (1968) |

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| 4 | Alonso et al. (1999) | Modified Cam-Clay | Roscoe and Burland (1968) |
| 5 | Oldecop and Alonso (2001) | Incremental rockfill plasticity | Oldecop and Alonso (2001) |
| 6 | Loret and Khalili (2002) | Drucker–Prager elastic–plastic | Drucker and Prager (1952) |
| 7 | Wheeler et al. (2003) | Modified Cam-Clay | Roscoe and Burland (1968) |
| 8 | Gallipoli et al. (2003) | Modified Cam-Clay | Roscoe and Burland (1968) |
| 9 | Chiu and Ng (2003) | NorSand critical-state sand | Jefferies (1993) |
| 10 | Tamagnini (2004) | Modified Cam-Clay | Roscoe and Burland (1968) |
| 11 | Sheng et al. (2004) | Modified Cam-Clay | Roscoe and Burland (1968) |
| 12 | Fredlund and Pham (2006) | Empirical volume–mass (no CS parent) | — |
| 13 | Sun et al. (2007) | Modified Cam-Clay | Roscoe and Burland (1968) |
| 14 | Li (2007) | Modified Cam-Clay (thermodynamic) | Roscoe and Burland (1968) |
| 15 | Gallipoli et al. (2008) | Modified Cam-Clay | Roscoe and Burland (1968) |
| 16 | Khalili et al. (2008) | Drucker–Prager elastic–plastic | Drucker and Prager (1952) |
| 17 | Sheng et al. (2008) | Modified Cam-Clay | Roscoe and Burland (1968) |
| 18 | Stropeit et al. (2008) | Modified Cam-Clay (anisotropic) | Roscoe and Burland (1968) |
| 19 | Zhang and Ikariya (2011) | Houlsby energy-based E-P | Houlsby (1990) |
| 20 | Zhou et al. (2012a, b) | Modified Cam-Clay | Roscoe and Burland (1968) |
| 21 | Liu and Muraleetharan (2012) | Drucker–Prager sand | Drucker and Prager (1952) |
| 22 | Lloret-Cabot et al. (2013) | Modified Cam-Clay | Roscoe and Burland (1968) |
| 23 | Ghasemzadeh and Amiri (2013) | Modified Cam-Clay | Roscoe and Burland (1968) |
| 24 | Wong and Mašín (2014) | Kolymbas hypoplastic | Kolymbas (1991) |
| 25 | Zhou and Sheng (2015) | Modified Cam-Clay | Roscoe and Burland (1968) |
| 26 | Zhou et al. (2015) | Bounding-surface plasticity | Dafalias and Popov (1975) |
| 27 | Tourchi and Hamidi (2015) | Modified Cam-Clay | Roscoe and Burland (1968) |
| 28 | Lloret-Cabot et al. (2017) | Modified Cam-Clay | Roscoe and Burland (1968) |
| 29 | Chong (2017) | Hardin–Drnevich cyclic backbone | Hardin and Drnevich (1972) |

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|---|---|---|---|
| 30 | Gholizadeh and Latifi (2018) | Modified Cam-Clay | Roscoe and Burland (1968) |
| 31 | Li and Yang (2018) | Modified Cam-Clay | Roscoe and Burland (1968) |
| 32 | Bruno and Gallipoli (2019) | Bounding-surface sand (Manzari–Dafalias) | Manzari and Dafalias (1997) |
| 33 | Cheng et al. (2020) | Prevost two-surface thermo clay | Prevost (1985) |
| 34 | Xiong (2020) | Large-strain elastic–plastic (Borja) | Borja and Tamagnini (1998) |
| 35 | Mahmoodabadi and Bryson (2021) | Modified Cam-Clay | Roscoe and Burland (1968) |
| 36 | Moghaddasi et al. (2021) | Bounding-surface plasticity | Dafalias and Popov (1975) |
| 37 | Tafili and Machaček (2023) | Kolymbas hypoplastic | Kolymbas (1991) |
| 38 | Lu et al. (2023) | Double-structure Cam-Clay | Alonso et al. (1999) |
| 39 | Corman (2023) | Modified Cam-Clay + gas | Roscoe and Burland (1968) |
| 40 | Sojoudi and Li (2023) | Modified Cam-Clay | Roscoe and Burland (1968) |
| 41 | Quevedo et al. (2024) | Modified Cam-Clay + compaction | Roscoe and Burland (1968) |
| 42 | Yang et al. (2024) | Kolymbas hypoplastic | Kolymbas (1991) |
| 43 | Kadivar et al. (2024) | Bounding-surface sand | Manzari and Dafalias (1997) |

### 3.1 Critical-state elasto-plastic lineage based on Modified Cam-Clay

Modified Cam-Clay (Roscoe and Burland 1968) supplies the stress–dilatancy relation, elliptical yield surface and logarithmic hardening law that underpin the Barcelona Basic Model (Alonso 1990) and its direct descendants: the double-structure expansive-clay model (Alonso 1999), the expansive-clay framework of Gens and Alonso (1992) and the extended Cam Clay formulation with hydraulic hysteresis proposed by Tamagnini (2004). Models that retain the Cam-Clay structure but incorporate temperature or chemistry, such as Tourchi and Hamidi (2015) and Lu et al. (2023), also belong to this lineage. Their unsaturated extensions modify the hardening law to include suction-dependent preconsolidation pressure and introduce scanning water-retention curves; no change is made to the Cam Clay flow rule.

### 3.2 State parameter frameworks derived from NorSand

The NorSand family, originally developed for saturated sand by Jefferies (1993), introduces a state parameter that measures the distance to the critical-state line and supplies a nonlinear dilatancy relation. This concept appears in unsaturated form in Chiu and Ng (2003), Zhou and Sheng (2015), and Li and Yang (2018). The saturated NorSand dilatancy equation is retained, but the critical-state line is shifted in the suction or saturation space; suction therefore alters dilatancy through its effect on the state parameter rather than through a separate empirical law.

### 3.3 Bounding surface plasticity roots

Bounding surface plasticity, formalised for saturated soils by Dafalias and Popov P1975) and later extended by Manzari and Dafalias (1997), is adopted, for instance, by Zhou et al. (2015) for small-strain unsaturated behaviour and by Bruno and Gallipoli (2019) for hydro-mechanical coupling. In these formulations, the radial mapping rule and the bounding surface evolution law remain unchanged from the saturated original; partial saturation is introduced through a suction-dependent shift of the bounding surface and, for Bruno and Gallipoli G2019), through a pair of hydraulic and mechanical bounding curves that capture double hysteresis.

### 3.4 Hypoplastic reference model

The hypoplastic model of Kolymbas (1991) for saturated soils provides the backbone for the partially saturated extensions by Wong and Mašín (2014) and Tafili and Mašín (2023). The stress rate equation and the intergranular strain concept are preserved, while suction-dependent scalar functions scale the stiffness moduli. In Tafili and Mašín, additional internal variables are introduced so that both hydraulic and mechanical hysteresis influence the hypoplastic modulus without altering the form of the reference hypoplastic equation.

### 3.5 Incremental plasticity and skeleton stress approaches

Oldecop and Alonso (2001) extend a granular rockfill model formulated incrementally for saturated conditions; only the hardening modulus is modified to include suction-dependent compressibility. The skeleton stress formulation of Loret and Khalili (2002) and Zhang and Ikariya (2011) stems from the concept proposed by Houlsby (1990) for saturated porous media. Partial saturation is introduced by selecting a work conjugate pair consisting of skeleton stress and degree of saturation, while the elastic plastic potential and flow rule remain identical to the saturated precursor.

### 3.6 Large strain and thermo-plastic origins

The finite strain model of Xiong (2020) follows the saturated large deformation framework of Borja and Koliji (2009); suction alters the effective stress, but no additional hydraulic variable is solved. Two surface plasticity for saturated clays (Prevost 1985) supplies the structure for the thermo-hydro-mechanical model of Cheng et al. (2020), where temperature shifts the bounding surfaces and suction follows a retention relation embedded in the hardening law.

### 3.7 Effective stress generalisation of Khalili and Khabbaz (1998)

The work conjugate effective stress proposed by Khalili and Khabbaz (1998) for saturated porous media defines the Bishop weighting factor $\chi$ as a suction-dependent function that approaches unity at the air entry value. This relation provides a closed-form link between suction and effective stress without requiring an explicit water retention curve. It has been adopted—sometimes in modified form—in the coupled models of Khalili et al. (2008), Liu and Muraleetharan (2012), and Mahmoodabadi and Bryson (2021). In these formulations, the yield surface, hardening law, and dilatancy rule follow the critical state structure of their respective saturated precursors, while the Khalili Khabbaz $\chi(S)$ relation supplies the additional suction dependency. When hysteresis is required, $\chi$ is evaluated on the wetting or drying branch of the scanning water retention curve, but the functional form remains that of the original saturated expression. The weighting function $\chi(S)$ proposed by Khalili and Khabbaz (1998) does not constitute a saturated constitutive model. Yet it has become the preferred link between matric suction and effective stress in many recent unsaturated formulations. Each of those formulations continues to rely on an established saturated parent framework for its yield surface, hardening law, and dilatancy rule; the Khalili–Khabbaz relation is inserted only to replace the constant Bishop coefficient. The principal examples are summarised in Table 3.

**Table 3.** Examples of implementation of χ(S) function in constitutive models.

| **Unsaturated formulation** | **Saturated parent retained** | **Role of χ(S) function** |
|---|---|---|
| Khalili et al. (2008) | Modified Cam-Clay | χ(S) substitutes Bishop's χ; a suction-rate term is added so that cyclic changes in suction feed back into plastic hardening. |
| Liu and Muraleetharan (2012) | Drucker–Prager sand model | χ(S) provides the suction-dependent component of the effective stress; yield and flow rules remain Drucker–Prager. |
| Mahmoodabadi and Bryson (2021) | Modified Cam-Clay | χ(S) is coupled with an internal water-retention equation, allowing simultaneous solution of stress, strain, and degree of saturation. |
| Tafili and Machaček (2023) | Kolymbas hypoplasticity | χ(S) scales the hypoplastic moduli; hydraulic and mechanical hysteresis are introduced through additional internal variables. |

In every case, the χ(S) expression enters only the stress calculation, leaving the parent yield geometry and plastic flow direction unchanged. By substituting a physically motivated, suction-dependent weighting factor for Bishop's constant, these authors extend the predictive range of their saturated models to partial saturation without altering their core mathematical structure.

In every case, the χ(S) expression enters only the stress calculation, leaving the parent yield geometry and plastic flow direction unchanged. By substituting a physically motivated, suction-dependent weighting factor for Bishop's constant, these authors extend the predictive range of their saturated models to partial saturation without altering their core mathematical structure.

## 4 Choice of Stress State Variables

### 4.1 Theoretical Formulations of Unsaturated Stress Variables

Choosing appropriate stress state variables in unsaturated soil mechanics is fundamental to extending classical effective stress concepts. One seminal approach is Bishop's (1959) effective stress (Eq. (1)), which augments Terzaghi's principle with a saturation-dependent factor. In Bishop's formulation, the effective stress is defined as the net stress (total stress minus pore-air pressure) plus a fraction of suction (pore-air minus pore-water pressure) scaled by an empirical parameter χ that depends on the degree of saturation.

$$\sigma' = (\sigma - u_a) + \chi\,(u_a - u_w) \qquad (1)$$

where $\sigma$ is the total stress, $u_a$ is the pore-air pressure $u_w$ the pore-water pressure, and χ is an effective stress parameter typically dependent on the degree of saturation.

When the soil is fully saturated (χ=1), this reduces to the standard Terzaghi effective stress; suction has no effect at zero saturation (χ=0). The Bishop stress concept elegantly ties unsaturated behavior to a single "effective" stress variable but introduces the need to define χ(S) for all saturation states. Determining a suitable χ–saturation function has proven challenging, as it must capture complex phenomena like hysteresis and microstructural changes during wetting and drying. This limitation means that while Bishop's stress provides a useful theoretical bridge from saturated to unsaturated conditions, its predictive success hinges on supplemental empirical relationships for χ.

An alternative framework is the dual-stress approach proposed by Fredlund and Morgenstern (1977), which treats net stress $(\sigma - u_a)$ and matric suction $(u_a - u_w)$ as two independent stress state variables. Rather than folding suction into an effective stress with a fitting parameter, this approach explicitly preserves the separate roles of air and water pressures. The dual-stress concept aligns neatly with thermodynamic principles by assigning each stress variable its own work-conjugate strain variable, thereby maintaining theoretical consistency without an arbitrary mixing parameter. Critical-state-based models in unsaturated soil mechanics readily adopted this two-variable description, allowing extensions of saturated constitutive laws into a suction-augmented stress space. For example, the Barcelona Basic Model (BBM) of Alonso et al. (1990) employs net stress and suction as primary variables to capture phenomena like wetting-induced collapse that single-stress models struggled with. A key advantage of the dual-stress formulation is that it bypasses the need for a Bishop-type χ function – suction itself enters the model directly, lending a clearer physical interpretation. However, using two independent stresses also means departing from the simplicity of a unifying effective stress. For practitioners accustomed to Terzaghi's principle, the dual-stress framework can be less intuitive, and it necessitates more complex constitutive surfaces (yield and failure criteria defined in an expanded stress–suction space). Despite this, the dual-variable approach has proven very effective in reproducing essential unsaturated soil behaviors, from shear strength increases due to suction to large volume changes upon wetting. Its explicit suction treatment set the stage for many modern models that handle unsaturated response by tracking how yield stress or hardening parameters evolve with changes in suction.

A third perspective comes from rigorous thermodynamic treatments of unsaturated porous media. Houlsby (1997) built on earlier work in saturated soils (Houlsby, 1979) to derive constitutive relationships based on energy conjugate pairs. In a rate of work analysis, he demonstrated that for unsaturated soil, the appropriate definition of stress must account not only for deformation of the soil skeleton but also for changes in water content. Houlsby's formulation yielded an effective stress expression that is mathematically consistent with Bishop's proposal; the degree of saturation directly enters the effective stress term. Crucially, however, it also identified that an additional independent variable (and associated strain-like variable) is required to capture the work done by suction changes. In other words, no single scalar stress can fully describe unsaturated behavior; one must include a separate thermodynamic conjugate for the degree of saturation (for example, a variable related to the volumetric water content). This theoretical insight bridges the gap between Bishop and Fredlund's approaches: it vindicates Bishop's effective stress as a valid stress measure (provided saturation is accounted for in the stress), but in the same breath insists on a second state variable akin to suction to satisfy energy conservation. Houlsby's work thus underpins later "saturation-based" formulations that explicitly build saturation or moisture-dependent terms into constitutive laws. Several researchers expanded on this concept by proposing composite stress variables or augmented effective stresses—e.g., tensorial forms combining net stress, suction, and saturation, all aiming to ensure that mechanical work and water retention energy are properly accounted for. In summary, there are multiple theoretical frameworks for unsaturated stress: Bishop's empirically enhanced effective stress, the dual (net stress and suction) description, and thermodynamically derived mixed formulations. No universal consensus has emerged on a single “best” stress variable; the preferred choice depends on the theoretical context of the model's theoretical context, the problem's nature, and the experimental data available for calibration. Each framework offers a different balance between simplicity and rigor, as further examined through its use in various constitutive models.

### 4.2 Comparative Analysis Across Constitutive Models

As summarized in Table 1, constitutive models in unsaturated soil mechanics have embraced the above stress definitions in different forms. Some models retain a Bishop-type effective stress as the primary variable, incorporating the degree of saturation into a single stress term, whereas others treat suction as an independent variable alongside net stress, and more recent formulations often introduce hybrid stress variables linked to saturation. This diversity reflects the trade-offs inherent in each choice. From a theoretical consistency and generality standpoint, the dual-stress approach offers broad applicability across the full range from dry to saturated states without modification – when suction vanishes at saturation, one simply recovers classical Terzaghi effective stress. Bishop-type formulations can also span from 0% to 100% saturation, but their generality is constrained by the accuracy of the chosen $\chi(S)$ function. Indeed, Jommi (2000) argued that suction affects soils in two distinct ways: (1) by directly contributing to mean stress (the effect Bishop’s stress captures) and (2) by inducing a stabilizing inter-particle force via water menisci. A single effective stress cannot represent both effects, so models strictly relying on Bishop’s $\sigma'$ may fail under certain conditions (e.g., at very low saturations where capillary bonding dominates). The introduction of a second stress variable or an internal state parameter is theoretically necessary to achieve a general description. Models like the BBM implicitly acknowledge this by expanding the yield surface in suction space, and others make it explicit: Gallipoli et al. (2003) introduce a suction-dependent bonding variable to complement Bishop’s stress for capturing capillary cohesion. Such additions enhance generality and physical realism, ensuring that the model can still represent the vanishing but stiff soil skeleton strengthened by dried menisci even as saturation approaches zero. Saturation-based frameworks (those derived with explicit inclusion of saturation, following Houlsby’s principles) are arguably the most theoretically consistent, as they treat the coupled mechanical and hydraulic energies in a unified way. For instance, Wheeler et al. (2003) use Bishop’s effective stress and a “modified suction” term as independent variables, allowing their model to reduce to correct limits at full saturation and account for unsaturated effects at all intermediate states gracefully. In their formulation, soil compressibility is not a fixed property but varies with saturation, interpolating between the behavior of a fully dry and a fully saturated soil. This yields a high degree of generality: the model inherently covers extremes by design, rather than requiring separate empirical tweaks in different regimes. In summary, when it comes to theoretical soundness and breadth of application, dual-stress and saturation-based formulations score highly due to their firm physical basis (actual pore pressures and degree of saturation) and built-in handling of limiting cases, whereas Bishop-type approaches rely on a single-variable approximation that may require augmentation for full generality.

The physical meaning of the chosen stress variables and parameters is closely related to the above considerations. The appeal of the dual-stress framework lies in using directly measurable quantities (net stress and suction) with clear physical interpretations: suction reflects matric tension in the pore fluid and its effect on inter-particle forces, while net stress is the external stress carried by the soil skeleton. Parameters introduced in dual-stress models (for example, those governing how yield or hardening evolves with suction) often correlate with tangible soil properties, even if they must be empirically determined. By contrast, Bishop's effective stress parameter χ has a less straightforward physical interpretation—it is sometimes viewed as the fraction of the soil's pore space that is water-filled or the fraction of the load carried by the soil's solid phase. Still fundamentally, χ is an abstract fitting function. As a result, models that rely on χ may introduce parameters lacking direct observables, making calibration more phenomenological. The newer saturation-based models strive to improve on this by linking parameters to soil-water retention behavior or microstructure. For example, Gallipoli's "bonding" variable is defined as a function of suction and degree of saturation, embodying a physical concept (capillary bonding) rather than a purely mathematical factor. Likewise, Wheeler et al.'s modified suction incorporates porosity, tying the hydraulic state to the soil structure. These additional variables enrich the physical meaning of the model at the expense of simplicity. In essence, dual-stress and advanced saturation-based approaches embed more physics into the constitutive description (each parameter or variable has a role grounded in soil behavior), whereas the Bishop-type approach is more empirical, condensing all unsaturated effects into one factor.

The differences between stress variable choices become even more pronounced when evaluating practical modeling aspects such as calibration effort, numerical implementation, and the ability to capture hysteresis and coupling. A single effective stress approach (Bishop-type) is often lauded for its numerical convenience: it allows unsaturated behavior to be introduced into existing constitutive models with minimal formal changes. One can take a well-calibrated model for saturated soil and simply replace the effective stress with Bishop's σ′; the same yield functions and flow rules can then be applied, using an estimated χ(S) to adjust for partial saturation. This convenience has made Bishop-type formulations popular in engineering practice for preliminary analyses. However, the calibration effort should not be underestimated: one must determine the χ function appropriate for the soil in question. In some cases, χ can be inferred from the soil-water characteristic curve or shear strength tests at various suctions (e.g., fitting the model to match the apparent cohesion intercepts in unsaturated triaxial tests). Researchers like Khalili and Khabbaz (1998) proposed universal forms for χ(S), but in practice, those still require validation against experimental data for each soil. Dual-stress models typically entail a higher calibration burden. Because suction is an independent variable, constitutive models must define how mechanical properties evolve as suction changes. The BBM, for instance, introduces a so-called loading-collapse (LC) yield curve that shifts upward in p–q space with increasing suction, and this shifting requires additional parameters (often obtained by fitting oedometer or triaxial tests under different constant suctions). Wheeler and Sivakumar's (1995) variant of the BBM demonstrated that careful empirical calibration of the compression curve at multiple suctions can improve model accuracy, but this means extensive testing (each suction level demands its calibration point). Saturation-based models share some of these demands: they usually require calibration of a soil-water retention model (to relate suction and saturation) and to the mechanical parameters. On one hand, having an explicit retention model can leverage existing laboratory measurements (water retention curves, drying–wetting cycles) to inform the mechanical model – for example, the retention curve hysteresis could guide the shape of the bonding function in Gallipoli's model. On the other hand, implementing features like distinct wetting and drying yield surfaces (as in Wheeler et al. 2003) means even more parameters (yield surface positions, slopes, etc., for each path) and more complex testing protocols to identify them. Thus, in terms of calibration effort, Bishop-type models might be considered "simpler" (there is only one main function to calibrate, albeit an important one), whereas dual-stress and saturation-based models demand broader datasets to pin down multiple functions.

The ability to handle hysteresis and coupled hydro-mechanical behavior is a critical benchmark for unsaturated soil models, and here, the more sophisticated stress variables show their strength. By itself, Bishop's original formulation, cannot distinguish wetting from drying. The parameter$\chi$ is typically assumed to follow a single-valued saturation function, neglecting the path dependence inherent in the soil–water retention process. If a model using Bishop's stress is to reproduce hysteresis, it must incorporate hysteretic scanning curves into the $\chi(S)$ relation or augment the model with additional rules, which complicates the originally simple framework. By contrast, a dual-stress approach naturally accommodates independent hydraulic modeling. One can couple a hysteretic soil-water retention curve to the mechanical model: suction at a given water content will differ on drying vs. wetting paths, and since suction is an explicit state variable, the mechanical response will indirectly reflect that difference. Modern dual-stress models explicitly build in hydraulic hysteresis. Wheeler et al. (2003) extended their critical-state model by introducing separate yield surfaces for wetting and drying (the so-called suction increase SI and suction decrease SD surfaces), ensuring that the plastic compression upon wetting follows a different trajectory than upon drying. This kind of built-in hysteresis is a hallmark of saturation-based formulations inspired by Houlsby's work. By including the degree of saturation (or a related internal variable) in the constitutive equations, these models directly couple changes in water content with changes in stiffness and strength. The result is a more faithful reproduction of cyclic drying–wetting behavior: for example, a Wheeler-type model can predict that a soil will exhibit a collapse deformation when wetting from a given suction, but much less swelling when drying back to that suction, reflecting the irreversible nature of pore structure changes. Regarding hydro-mechanical coupling, having suction or saturation as a state variable is essential for fully coupled analyses (e.g., finite element simulations of transient infiltration). Dual-stress models interface well with flow equations – suction evolves according to the flow regime, and the mechanical model responds to the updated suction at each time step. Saturation-based models often go one step further, integrating the retention law into the constitutive model so that volumetric deformations can influence the retention behavior (changing porosity and thus the SWRC). This bidirectional coupling (sometimes called hydraulic–mechanical coupling) is necessary to capture phenomena like consolidation under changing moisture or air-entry during collapse. It is achievable in dual-stress formulations (by updating a separate retention model based on computed volume change), but is more naturally accounted for in frameworks derived with thermodynamic consistency. The trade-off is that these sophisticated models are numerically more complex. Handling multiple yield surfaces or additional internal variables (such as bonding or modified suction) increases the computational effort for constitutive integration and may require smaller time steps or more robust solvers. Early-generation models with fewer variables were simpler to implement and more robust in numerical simulations, whereas advanced models demand careful coding and sometimes special numerical treatments for convergence. Fortunately, improvements in computational algorithms and computing power have been mitigating this issue. As noted in recent reviews, the trend in unsaturated soil modeling has been toward incorporating more physics (and hence more variables) into constitutive models, accepting the higher complexity as the price for improved realism. In practice, the choice of stress state variables often reflects this compromise: if the project at hand requires capturing hysteresis, anisotropy, or precise coupling (e.g., in cyclic infiltration problems or engineered barrier systems), a model with dual-stress or saturation-based variables is warranted. If instead a simpler approximation suffices (e.g., an order-of-magnitude estimate of settlement due to wetting), a Bishop-type effective stress model calibrated with a few data points might be preferred for its ease of use.

## 5 Critical Components in Constitutive Model Formulation

Transitioning from saturated to unsaturated conditions in constitutive modeling involves a series of modifications to capture the influence of suction and partial saturation. Below are core elements that are often adapted or redefined in unsaturated models.

### 5.1 Yield and Failure Criteria

A primary question is how to incorporate suction into yield and failure surfaces. Many models build on classical failure criteria like Mohr-Coulomb or Drucke-Prager by introducing a suction-dependent term into the shear strength equation (Fredlund and Rahardjo, 1993). Advanced approaches embed suction directly into plastic yield surfaces, expanding the conventional stress space to include either suction or degree of saturation (Alonso et al., 1990; Wheeler et al. (2003)). This enables more accurate collapse modeling, and irreversible volume changes when soils transition between wetting and drying cycles.

### 5.2 Hardening Laws and Hysteresis

In unsaturated soil models, hardening laws typically depend on plastic shear strain (or volumetric strain) and suction changes. For instance, when suction decreases (e.g., due to infiltration), the soil may experience rapid collapse or softening. Typically handled through a plastic volumetric strain formulation tied to suction changes (Oldecop and Alonso (2001)). Many formulations use an additional hardening or softening rule tied to suction to adjust the yield surface, reflecting how partial saturation can strengthen or weaken the soil skeleton (Sheng et al. (2008)).

Soils often exhibit hysteresis in their soil–water retention curve (SWRC). Hysteresis arises because the path (wetting versus drying) affects how pore water redistributes. Modern constitutive models thus include hysteresis in mechanical and hydraulic components to accurately replicate phenomena such as cyclic wetting–drying and the associated volume change (Tarantino and Tombolato (2005)). Early theoretical developments by Mualem (1974) and later practical models proposed by Pham et al. (2003), Fredlund and Pham (2006) have provided methodologies to capture both drying and wetting paths. These hysteresis models are essential for accurately predicting the cyclic behavior of unsaturated soils, particularly under repeated wetting and drying conditions.

### 5.3 Coupling with the Soil–Water Characteristic Curve

A critical link exists between mechanical behavior and the soil–water characteristic curve. Changes in suction directly modify water retention, which alters effective stress and stiffness. Some models couple a hydraulic sub-model (e.g., Van Genuchten (1980); Fredlund and Xing (1994)) with a mechanical sub-model, jointly solving for both stress-strain and water retention behaviors. This coupling is crucial when evaluating processes like infiltration, consolidation, or drying-induced cracking, as all are driven by suction variations over time. Furthermore, modern constitutive formulations must account for anisotropic effects and complex coupling between hydraulic and mechanical processes. Research by Wheeler et al. (2003) and subsequent studies have demonstrated that anisotropy in soil fabric can lead to directional dependencies in the evolution of yield surfaces. In response, advanced models now feature yield surfaces whose shapes and orientations evolve with both stress history and changes in suction. The integration of these aspects is complemented by energy-based approaches that capture the interplay between plastic deformation and hydraulic hysteresis, as highlighted in the works of Dangla et al. (2002).

### 5.4 Parameter Identification and Calibration

Because unsaturated models typically require more parameters (e.g., those describing suction hardening, SWRC shape, or hysteresis laws), specialized experimental data are needed. Suction-controlled triaxial, oedometer, or shear box tests help calibrate these additional parameters. Moreover, advanced methods like inverse analysis or machine learning are increasingly used to optimize parameter sets, particularly in large-scale or highly nonlinear problems (Zhang et al. (2021)).

Effective unsaturated soil modeling demands close attention to the interplay of suction, degree of saturation, and mechanical variables. Yield criteria, hardening laws, and hysteresis interact in ways that do not appear in saturated soil models, making careful calibration and robust coupling to hydraulic models essential.

## 6 Constitutive Models for Unsaturated Soils: Categories and Developments

Several constitutive models have been proposed to capture unsaturated soils's mechanical and hydraulic complexities. These models can be broadly categorized based on their theoretical underpinnings, choice of stress variables, coupling strategies, and consideration of phenomena such as anisotropy, hysteresis, and small-strain behavior. The following sections provide an overview of key model categories, tracing their historical evolution and highlighting representative examples.

### 6.1 Early Models

#### 6.1.1 Foundations and Extension from Saturated Frameworks

Early efforts to describe unsaturated soil behavior often began with empirical extensions of saturated soil models. However, these simplified methods struggled to capture important phenomena such as wetting-induced collapse.

A significant turning point emerged when Fredlund and Morgenstern (1977) proposed two independent stress variables, i.e., net stress and suction, thereby circumventing the need to force suction into a single effective stress parameter. This shift aligned closely with contemporary advances in critical state soil mechanics (Roscoe and Burland (1968)), motivating researchers to include suction in elastoplastic frameworks.

Among the most influential early models is the Barcelona Basic Model (BBM), introduced by Alonso et al. (1990). The BBM explicitly incorporates suction as an independent state variable within an elastoplastic framework, enabling the prediction of key behaviors like collapse upon wetting and irreversible volumetric changes. It represented a substantial leap by extending concepts from the Modified

Cam-Clay model to unsaturated conditions. Irreversible volume changes can be attributed to the elastic expansion and compression of micro-pores, which in turn define the plastic volumetric change of the macro-pores. This concept was operationalized by introducing two additional yields referred to as the SI and SD yield curves, illustrated in Figure 1.Despite its success, the original BBM did not fully address anisotropic behavior or certain microstructural effects, prompting subsequent refinements. Wheeler and Sivakumar (1995) proposed a model that retained the core BBM concepts but included fewer simplifying assumptions, improving predictions of stress-strain evolution across varying suction levels. Unlike the BBM, where normal consolidation lines were often assumed or derived from limited data, Wheeler and Sivakumar (1995) calibrated these lines empirically for different suction values. They observed that the change in the critical state line in the $q-p'$ plane (where q is deviatoric sterss and $p'$ is the mean net stress) is not directly proportional to suction; rather, it is governed by a suction function analogous to that assumed in the BBM. Other researchers, such as Gens and Alonso (1992), explored multi-mechanism and hierarchical approaches, paving the way for models capable of capturing complex wetting–drying cycles in expansive clays.

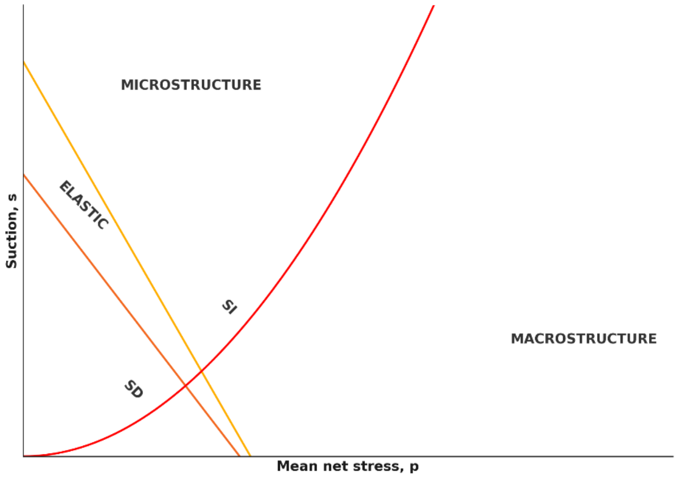

**Figure 1.** The geometric yield surface for a dual structure elastoplastic model (Alonso et al., 1990).

### 6.1.2 Anisotropy and Microstructural Refinements

As experimental evidence grew, studies revealed that many unsaturated soils exhibit anisotropic behavior, which standard isotropic models could not fully capture (Wheeler et al. (2003)). In response, modifications were introduced to rotate or reshape yield surfaces in stress-strain-suction space, thus reflecting the soil's inherent fabric (Cui and Delage (1996); D'Onza et al. (2010)). This rotation causes the constant suction cross-sections of the yield surface in the $q-p'$ plane to become inclined. Recognizing that many natural unsaturated soils exhibit anisotropic behavior not fully captured by isotropic models, Stropeit et al. (2008) integrated features of the S-CLAY1 model, originally proposed by Wheeler et al. (2003) for soft saturated clays with anisotropy, into the conventional BBM framework. In this enhanced model, an independent anisotropy parameter is introduced; however, unlike the approach of Cui and Delage, in which anisotropy is assumed constant concerning plastic strain advancement,Comparative simulations using the anisotropic model and the conventional BBM on experimental data from Cui and Delage (1996) revealed that the anisotropic formulation more accurately reproduced the experimental yield surfaces and volumetric strain evolutions under constant suction conditions. Building on this development, D'Onza et al. (2010) later modified the yield surface equation within the anisotropic model, resulting in an updated version that improved the model's fit to observed data.

Research on expansive minerals also highlighted the need to model separate macro- and microstructural mechanisms. Unsaturated soils containing highly expansive minerals, such as montmorillonite, exhibit markedly different behavior than soils with low or moderate expansiveness, particularly during the initial wetting process. In these materials, a significant irreversible swelling component is observed. For example, Chu (1973) conducted a series of wetting and drying cycles on highly expansive clay and documented a pronounced irreversible swelling during the first wetting event. The conventional BBM does not adequately predict such behavior. To address these limitations, subsequent studies by Gens and Alonso (1992), Alonso et al. (1994), and Alonso et al. (1999) introduced modified versions of the BBM specifically designed to capture irreversible volumetric changes during wetting-drying cycles in highly expansive soils. These enhancements allow for more accurate predictions of irreversible swelling and shrink-swell cycles, particularly in highly plastic montmorillonite clays.

Early unsaturated models laid the groundwork by integrating suction into elastoplastic frameworks and refining classical failure criteria. The BBM, its derivatives, and dual-stress formulations remain cornerstones that influence modern constitutive approaches.

## 6.2 Hysteresis strategies in constitutive modelling

Partial saturation introduces two distinct forms of hysteresis that must be represented if constitutive predictions are to remain reliable during repeated wetting–drying or cyclic loading:
(i) hydraulic hysteresis of the soil–water retention curve and (ii) mechanical hysteresis of the stress–strain response under reversals of suction or deviatoric loading.

The reviewed formulations fall into three broad strategies, each reflecting a different balance between physical transparency and numerical economy which are presented in Table 4.

The first strategy, employed in the Barcelona Basic Model group and many Cam Clay derivatives, treats mechanical behaviour with a single elasto-plastic surface while reproducing hysteresis exclusively through scanning branches of the water retention curve. During a drying path, the suction increment is mapped onto the drying main curve; on wetting, the path follows a family of scanning curves that converge towards the wetting main branch. Because suction is an independent state variable, the plastic hardening law shifts the yield surface in stress space according to the current suction, so hydraulic hysteresis indirectly influences stiffness and strength without modifying the mechanical yield function. This approach is straightforward to implement, provided that the drying and wetting branches are available from laboratory data, but it cannot capture mechanical ratcheting under deviatoric cyclic loading.

The second strategy introduces explicit mechanical hysteresis through kinematic or bounding surface hardening while retaining hydraulic scanning curves for the retention behaviour. Wheeler et al. (2003) superimposed two additional yield surfaces, SI and SD, which become active during suction increase and decrease respectively; plastic compression on wetting therefore follows a different trajectory from that on drying. Bruno and Gallipoli (2019) adopted a double-bounding surface framework: one surface governs hydraulic softening, the other mechanical cyclic degradation. The position and size of each bounding surface evolve with suction, allowing coupled hydraulic–mechanical ratcheting to emerge naturally. Such models reproduce the irreversible volumetric strains observed in cyclic wetting–drying tests, yet demand more internal variables and computational effort.

The third strategy embeds hysteresis directly in the material stiffness, utilizing additional internal variables that track the hydraulic path. Hypoplastic extensions by Wong and Mašín (2014) and Tafili and Machaček (2023) assign two memory variables to each integration point: the current suction and the suction at the last reversal. These variables modify the hypoplastic modulus and pseudo-Poisson ratio, so that unloading and reloading curves differ without requiring separate yield surfaces. A similar idea appears in saturation-based formulations derived from Houlsby's thermodynamic framework: the degree of saturation serves as a history parameter, and hysteresis is recovered by defining distinct wetting and drying evolution equations for the saturation variable. The principal attraction of these schemes is numerical robustness; the trade-off is that the physical meaning of the added parameters can be opaque, complicating calibration.

**Table 4.** Hysteresis representation in unsaturated constitutive models.

| # | Model | Hydraulic hysteresis | Mechanical hysteresis | Implementation strategy* |
|---|---|---|---|---|
| 1 | Alonso et al. (1990) | Yes – scanning SWRC | No | H-only |
| 2 | Gens and Alonso (1992) | Yes – scanning SWRC | Yes – swell–shrink | H + M (kinematic shift) |
| 3 | Wheeler and Sivakumar (1995) | No | No | none |
| 4 | Alonso et al. (1999) | Yes – scanning SWRC | No | H-only |
| 5 | Oldecop and Alonso (2001) | No | No | none |
| 6 | Loret and Khalili (2002) | No | No | none |
| 7 | Wheeler et al. (2003) | Yes – scanning SWRC | Yes – kinematic SI/SD | H + M (dual yield) |
| 8 | Gallipoli et al. (2003) | Yes – scanning SWRC | No | H-only |
| 9 | Chiu and Ng (2003) | No | No | none |
| 10 | Tamagnini (2004) | Yes – scanning SWRC | No | H-only |
| 11 | Sheng et al. (2004) | Yes – scanning SWRC | No | H-only |
| 12 | Fredlund and Pham (2006) | No | No | none |
| 13 | Sun et al. (2007) | Yes – scanning SWRC | No | H-only |
| 14 | Li (2007) | No | No | none |
| 15 | Gallipoli et al. (2008) | Yes – scanning SWRC | No | H-only |

| | | | | |
|---|---|---|---|---|
| 16 | Khalili et al. (2008) | Yes – scanning SWRC | Yes – suction-rate cyclic | H + M (rate dep.) |
| 17 | Sheng et al. (2008) | No | No | none |
| 18 | Stropeit et al. (2008) | No | No | none |
| 19 | Zhang and Ikariya (2011) | No | No | none |
| 20 | Zhou et al, (2012a, b) | Yes – scanning SWRC | No | H-only |
| 21 | Liu and Muraleetharan (2012) | Yes – scanning SWRC | No | H-only |
| 22 | Lloret-Cabot et al. (2013) | Yes – scanning SWRC | No | H-only |
| 23 | Ghasemzadeh and Amiri (2013) | No | No | none |
| 24 | Wong and Mašín (2014) | Yes – implicit | Minor implicit | Embedded memory |
| 25 | Zhou and Sheng (2015) | Yes – scanning SWRC | No | H-only |
| 26 | Zhou et al. (2015) | No | No | none |
| 27 | Tourchi and Hamidi (2015) | No | No | none |
| 28 | Lloret-Cabot et al. (2017) | Yes – scanning SWRC | No | H-only |
| 29 | Chong (2017) | No | Yes – Masing | M-only |
| 30 | Gholizadeh and Latifi (2018) | Yes – scanning SWRC | No | H-only |
| 31 | Li and Yang (2018) | Yes – scanning SWRC | No | H-only |
| 32 | Bruno and Gallipoli (2019) | Yes – twin SWRC bounding | Yes – mechanical bounding | H + M (double bounding) |
| 33 | Cheng et al. (2020) | Yes – thermal–hydraulic loops | Minor thermal | H-only (thermal) |
| 34 | Xiong (2020) | Yes – scanning SWRC | No | H-only |
| 35 | Mahmoodabadi and Bryson (2021) | Yes – scanning SWRC | No | H-only |
| 36 | Moghaddasi et al. (2021) | Yes – scanning SWRC | Yes – bounding surface and bonding | H + M (double bounding) |
| 37 | Tafili and Machaček (2023) | Yes – scanning SWRC | Yes – kinematic vars. | Embedded memory |
| 38 | Lu et al. (2023) | Yes – scanning SWRC | No | H-only |
| 39 | Corman (2023) | Yes – gas retention loops | Minor gas | H-only (gas) |
| 40 | Sojoudi and Li (2023) | No | No | none |
| 41 | Quevedo et al. (2024) | Yes – scanning SWRC | No | H-only |

| | | | | |
|---|---|---|---|---|
| 42 | Yang et al. (2024) | Yes – structural collapse scanning | Implicit (hypoplastic) | Embedded memory |
| 43 | Kadivar et al. (2024) | Yes – scanning SWRC | Yes – bounding surface | H + M (bounding) |

## 6.3 Models with Alternative Stress State Variables and Coupled Multi-Physics Interactions

In unsaturated soil mechanics, capturing the mechanical behavior of soils under varying saturation conditions requires not only the use of refined stress state variables but also the incorporation of coupled hydro-mechanical (and, in some cases, thermo- and chemically active) processes. In many constitutive formulations, suction or a function of suction is employed as the second stress variable. To predict mechanical behavior at the stress point level, the mechanical model must be solved simultaneously with a water retention model that governs variations in the degree of saturation (Fredlund and Rahardjo (1993)). This integration is essential because the water retention behavior directly influences the effective stress formulation (e.g., Bishop's stress, Eq. (1)). Yet, most models do not fully capture all features that arise from the complete coupling of mechanical and water retention responses.

While the BBM established a framework using net stress and suction as independent variables, subsequent research sought to refine the representation of the stress state in unsaturated soils by introducing alternative formulations. Alternative approaches have sought to unify or modify these definitions. For instance, Bishop's stress (Bishop, 1959) can be adapted by introducing a saturation-dependent parameter $\chi$. Other researchers propose tensorial stress parameters that combine net stress, suction, and degree of saturation (Kohgo et al. (1993); Kohgo et al. (1993); JOMMI and DI PRISCO (1994); Modaressi and Abou-Bekr (1994); Pakzad (1995);, Bolzon et al. (1996); Khalili and Khabbaz (1998)).

### 6.3.1 Coupling Mechanical and Water Retention Models

Fundamental contributions by Wheeler and Sivakumar (1995) established an elastoplastic critical state framework that laid the groundwork for later coupled formulations. Their work was subsequently enhanced by Wheeler et al. (2003) and Sheng et al. (2004), who explicitly introduced hydraulic hysteresis into the formulation. These models could reproduce complex phenomena such as collapse upon wetting and cyclic behavior under varying environmental conditions (Wheeler et al., 2003; Khalili et al. (2008)). For example, Cunningham et al. (2003) examined the coupled response of reconstituted unsaturated silty clay under hydraulic and mechanical loading. Building on this foundation, several advanced hydro-mechanical formulations have emerged. Ghasemzadeh and Amiri (2013) proposed a rate-independent elastoplastic model that integrated suction and degree of saturation into the hardening law, enabling simulation of collapse behavior under isotropic loading. Similarly, Gholizadeh and Latifi (2018) developed a fully coupled hydro-mechanical model that captures suction dependency in both shear strength and stiffness, offering improved predictions under varying saturation paths. Liu and Muraleetharan (2012) introduced a two-phase porous media model formulated within a critical state elastoplastic framework for sands and silts, incorporating the degree of saturation directly in the hardening rules, thus enabling the simulation of complex cyclic and transient loading paths. More recently, Zhou and Sheng (2015) proposed a state-dependent bounding surface model that captures the effects of suction and initial density, enabling more accurate predictions under loading–unloading conditions and accounting for compaction effects. Sun et al. (2007) extended coupled models by integrating density-dependent parameters to reflect microstructural effects in compacted soils, while Sun and Sun (2012) emphasized the role of matric suction in expansive clays by coupling swelling potential and suction variations. Lu et al. (2023) introduced a hydro-mechanical model capable of capturing a wide saturation range and explicitly considering the influence of the degree of saturation on stiffness and strength through novel saturation-dependent yield surfaces. Mahmoodabadi and Bryson (2021) contributed a fully coupled hydro-mechanical model capable of capturing hysteresis in the soil–water characteristic curve and simulating transient infiltration effects. Additionally, Li and Yang (2018) addressed overconsolidation effects by proposing a hydromechanical model that incorporates different yield surfaces based on the soil's consolidation state. Tourchi and Hamidi (2015) extended the critical state framework by coupling thermal effects with suction and stress, allowing for the simulation of unsaturated clays under varying temperatures. Xiong (2020) developed a three-dimensional elastoplastic model, further advancing the simulation of hydro-mechanical responses. Tafili and Machaček (2023) introduced a hypoplastic model with generalized hydro-mechanical coupling, effectively capturing the collapse behavior and stiffness evolution in unsaturated soils under varying degrees of saturation and stress paths. Quevedo et al. (2024) further refined coupled frameworks by accounting for compaction-induced changes in soil hydraulic properties, reinforcing the importance of initial state variables in hydro-mechanical modeling.

Several studies have focused on integrating water retention models directly into the mechanical formulation. JOMMI and DI PRISCO (1994) proposed a model in which the effect of suction is introduced by modifying the effective stress through Bishop’s stress tensor and incorporating a stabilizing force at inter-particle contacts to simulate the influence of crescent-shaped water bridges. In their model, yield stresses depend on the degree of saturation, and a unique water retention relationship between the degree of saturation and suction is employed. However, the absence of hydraulic hysteresis in their water retention formulation limits the model’s ability to capture the coupled behavior fully. Building on this framework, Tamagnini (2004) combined Jommi (2000) prposal with a modified Cam-Clay approach using Bishop’s stress and an additional hardening function dependent on the degree of saturation, with the water retention model of Romero and Vaunat (2000) to achieve a coupled mechanical and water retention model that captures bidirectional coupling.

Other unified frameworks have also been developed. Vaunat et al. (2000) proposed an elastoplastic hydro-mechanical model based on net stress and suction that couples the water retention model of Romero and Vaunat (2000) with the Barcelona Basic Model (BBM) for mechanical behavior. Their approach, however, results in a unidirectional coupling in which the influence of water retention on mechanical behavior is not fully accounted for. Loret and Khalili (2002) presented a coupled hydro-mechanical behavioral model in the critical state framework using the effective stress variable proposed by Khalili and Khabbaz (1998). Jommi (2000) further argued that suction has two distinct effects on the mechanical behavior at the macroscopic level: the modification of the mean stress (captured by Bishop’s stress) and a stabilizing effect at inter-particle contacts due to crescent-shaped water bridges-effects that a single stress state variable cannot fully model. Gallipoli et al. (2003) and Gallipoli et al.

(2008) introduced an elastoplastic framework that incorporates a bonding factor (a scalar variable dependent on suction and degree of saturation) to represent the stabilizing influence of water bridges, validating their approach with experimental data from various researchers. Similarly, Wheeler et al. (2003) proposed a unified coupled mechanical–water retention framework based on alternative stress variables. The primary variable is Bishop's stress tensor, and a second variable (modified suction) is introduced to capture additional hydraulic effects. This modified suction is defined as Eq. (2).

$$s^* = n\,(u_a - u_w) \tag{2}$$

where, n is the porosity.

Their formulation defines three yield surfaces, namely loading-collapse (LC), suction increase (SI), and suction reduction (SD), which together govern both mechanical and hydraulic responses. Following Houlsby (1997), the coupled evolution of strain and degree of saturation is integrated into the model. In this framework, the modified suction variable, in combination with Bishop's stress, serves as the input work variable for unsaturated soils. Their approach further distinguishes three yield surfaces for a given stress state: the loading–unloading collapse (LC) surface, the suction increase (SI) surface, and the suction reduction (SD) surface. Lloret-Cabot et al. (2013) and Lloret-Cabot et al. (2017) extended these models to three-dimensional stress conditions and non-proportional loading paths.

Other contributions in this category include frameworks developed by Li (2005), Li (2007), and Li (2007), which integrate thermodynamic concepts and multiphase considerations to model the coupling between the soil skeleton and water retention behavior. Zhou et al. (2012) and Zhou et al. (2012) proposed new volumetric strain equations and yield surfaces defined in stress-saturation space to model the nonlinear compressibility of soils under constant suction accurately. Their formulation assumes that the soil compressibility index is a function of the effective degree of saturation interpolated from the compressibility in the fully saturated and fully dry states. Eqs. (3) to (5) explain the model formulation for volumetric behavior.

$$v = N - \lambda(S_e)\boldsymbol{ln p'} \tag{3}$$

and

$$-dv = \lambda(S_e)\frac{dp'}{p'} + \frac{\partial\lambda(S_e)}{\partial S_e} ln\, p'\; dS_e \tag{4}$$

with

$$\lambda(Se) = \lambda_0 - (1 - S_e)^{a_1}(\lambda_0 - \lambda_d) \tag{5}$$

where, N is the intercept of the normal consolidation line in the saturated state, $\lambda(Se)$ is the slope in the unsaturated state as a function of the effective degree of saturation $(Se)$, $p'$ is Bishop's effective stress, $\lambda_0$ is the slope in the saturated state, $\lambda_d$ is the slope in the completely dry state, and $a_1$ is a parameter defining the influence of $Se$ on the compressibility.

Furthermore, Zhou et al. (2012) and Zhou et al. (2012) conceptually defined the effective degree of saturation changes by Eqs. (6) and (7).

$$dSe = \frac{\partial S_e}{\partial s}\, ds + \frac{\partial S_e}{\partial \varepsilon_{v\sigma}}\, d\varepsilon_{v\sigma} \tag{6}$$

where,

$$D = \frac{\partial S_e}{\partial \varepsilon_{v\sigma}} \tag{7}$$

and $\varepsilon_{v\sigma}$ represents the volumetric strain due to net stress changes.

Beyond standard hydro-mechanical coupling, specialized models have emerged employing bounding surface plasticity theory. Zhou et al. (2015) proposed a boundary surface model for unsaturated soils based on the state-dependent elastoplastic model of Chiu and Ng (2003), enhanced through coupling with a water retention model as proposed by Gallipoli et al. (2003).

Bruno and Gallipoli (2019) developed a coupled bounding surface model for isotropic stress conditions, combining a hydraulic law (which relates the degree of saturation to a scaled suction), presented in Eqs. (8) to (12) with mechanical law regarding a scaled stress variable presented in Eqs. (13) to (17).

$$\bar{S} = (s \times e)^{\frac{1}{\lambda_s}} \tag{8}$$

$$(S_r)_d = \left(1 + \left(\frac{s \times e^{1/\lambda_s} + C_d}{\omega_d^{\beta_d}}\right)^{0.7\frac{\lambda_s}{\beta_d \times m_d}}\right)^{-m_d} \tag{9}$$

$$(S_r)_w = \left(1 + \left(\frac{s \times e^{1/\lambda_s}{}^{\beta_w}}{\omega_w^{\beta_w} \times (1 + C_w \times s \times e^{1/\lambda_s}{}^{\beta_w})}\right)^{0.7\frac{\lambda_s}{\beta_w \times m_w}}\right)^{-m_w} \tag{10}$$

where, $C_d$ and $C_w$ are constants of integration which are presented as Eqs. (11) and (12).

$$C_d = \omega_d^{\beta_d}\left(S_r^{0.8\left(\frac{1}{m_d}\right)} - 1\right)^{0.7\frac{\beta_d \times m_d}{\lambda_s}} - s \times e^{1/\lambda_s}{}^{-\beta_d} \tag{11}$$

$$C_w = \frac{1}{\omega_w^{\beta_w}} \times \left(S_r^{0.8-\left(\frac{1}{m_w}\right)} - 1\right)^{0.8\frac{\beta_w \times m_w}{\lambda_s}} - 1\frac{1}{s \times e^{1/\lambda_s}{}^{-\beta_w}} \tag{12}$$

Like the hydraulic law, this mechanical law has two closed-form equations for decreasing and increasing values of scaled stress. Because of the mathematical form of scaled stress (Eq.(13)), there are two possible ways for producing a loading path: (1) by an increase of mean average skeleton stress ($p'$), and (2) by reduction of the degree of saturation. A similar argument exists for producing an unloading path (i.e., a decrease in mean average skeleton stress and an increase in the degree of saturation). The mathematical expressions for mechanical law are presented in Eqs. (13) to (17).

$$\bar{p} = p' \, S_r^{\frac{\lambda_r}{\lambda_p}} \bar{p} \tag{13}$$

$$e_l = \left(\left(\frac{\bar{p}}{\bar{p}_{ref}}\right)^{\gamma} + C_l\right)^{-\frac{\lambda_p}{\gamma}} \tag{14}$$

$$e_u = \frac{C_u}{\bar{p}^{\kappa}} \tag{15}$$

$$C_l = e_0^{\frac{-\gamma}{\lambda_p}} - \left(\frac{\bar{p}_o}{\bar{p}_{ref}}\right)^{\gamma} \tag{16}$$

$$C_u = e_0 \bar{p}_0^{\kappa} \tag{17}$$

where, $\bar{p}_{ref}$ is a reference scaled stress (corresponding to a void ratio equal to one), $\gamma$ is a material parameter, $C_l$ is the constant of integration for the loading path, $e_0$ and $\bar{p}_0$ are known values of void ratio and scaled stress, respectively, $C_u$ is the constant of integration on the unloading path, and $\kappa$ is the slope of the swelling line.

For overconsolidated soils, this model assumes that as the soil state approaches the unified normal consolidation line, the slope of the loading curve tends toward that of the line. This model requires only a few additional parameters beyond those used for saturated soils.

These advances have been complemented by extensive experimental investigations (e.g., Sivakumar (1993); Sharma (1998); Vassallo et al. (2007); Raveendiraraj (2009); Biglari et al. (2012)) that provide valuable data for validating the numerical predictions of coupled models. These contributions demonstrate the evolution from early models introducing suction as an independent variable to more advanced formulations capturing bidirectional coupling between mechanical deformation and water retention behavior.

### 6.3.2 Extending Coupling Beyond Hydro-Mechanical Interactions

As the understanding of unsaturated soils has evolved, it has become clear that many phenomena cannot be captured by considering only the mechanical and water retention interactions. Modern models increasingly incorporate multi-physics couplings, including thermal and chemical effects, to simulate field conditions more realistically. Early coupled models focused primarily on hydro-mechanical interactions, reproducing behaviors such as collapse on wetting paths. These models laid the foundation for subsequent efforts recognizing soil behavior's multidisciplinary nature.

For example, thermo-hydro-mechanical models have been developed to address nuclear waste disposal and geothermal energy extraction applications, where temperature variations significantly alter soil behavior. Similarly, chemically active soils subject to processes like contaminant transport and mineral dissolution require additional coupling to account for changes in microstructure and water retention (Dangla et al. (2002);, Abed and Sołowski (2019)).

Recent research has extended the coupled framework beyond purely hydro-mechanical interactions to encompass additional environmental and chemical processes. Studies by Gao et al. (2019) investigated the hydro-mechanical behavior of unsaturated soils over a wide suction range, emphasizing the importance of the initial state on coupled responses. In parallel, research by Ghasemzadeh and Amiri (2013) highlighted the role of rate-independent mechanisms and the need to integrate flow-deformation processes in saturated and unsaturated states. Comprehensive THMC formulations (Abed and Sołowski (2017); Abed and Sołowski (2019)) further enhance predictive capability in scenarios like nuclear waste disposal and geothermal applications. While these models capture a wider range of phenomena, their complexity increases considerably, raising challenges in parameter identification and numerical implementation.

Environmental factors play a significant role in coupled behavior. Bryson and Ahmed (2019) demonstrated that transient moisture fluctuations under rainfall critically affect slope stability, necessitating models that account for simultaneous mechanical and hydraulic responses. Similarly, thermal effects have been incorporated into some models.

### 6.4 Models Considering Clay-Bound Water

In the constitutive modeling of unsaturated soils, especially expansive clays, the role of clay-bound water has gained increasing attention due to its significant impact on mechanical and hydraulic behavior. Traditional models often treat water in soils as a homogeneous phase, yet clays, particularly smectite-rich varieties such as montmorillonite—retain structured layers of adsorbed water with distinct physicochemical properties.

Low (1979) provided foundational insights into the nature of water in montmorillonite-water systems, demonstrating that adsorbed water is not merely a thinner version of bulk water, but exhibits unique viscosity, dielectric, and mobility characteristics. This structured water, often referred to as "bound water," is retained tightly within the interlayer spaces of clay minerals and significantly affects swelling behavior, suction responses, and permeability.

Building on this, Sposito and Prost (1982) explored the molecular structure of water adsorbed on smectites, confirming the presence of discrete water layers oriented by electrostatic interactions with charged clay surfaces. These structured layers influence the energy state of water in the soil, leading to deviations from classical soil–water retention relationships (SWRCs) that assume bulk water behavior. Their findings underscored the need to differentiate between physically bound, capillary, and free water when modeling retention and deformation behavior in fine-grained soils.

Recognizing these complexities, Jacinto et al. (2012) experimentally demonstrated that variations in water density—induced by the presence of bound water, significantly shift the soil–water retention curve in expansive clays. This implies that standard retention models, which assume constant water density, may overestimate suction for a given degree of saturation. Their findings support the necessity of incorporating variable water density and bound water effects into constitutive formulations, especially for high-plasticity soils.

More recently, Sojoudi and Li (2023) proposed a thermodynamically consistent elastoplastic model for saturated clayey soils that explicitly accounts for bound water dehydration. Their approach integrates microstructural considerations by dividing the total water content into free and bound water, the latter governed by temperature and suction-dependent dehydration mechanisms. By including bound water effects, the model can more accurately predict thermal collapse and suction-induced volume changes, addressing behaviors inadequately captured by traditional frameworks.

Collectively, these contributions point to the critical need for constitutive models that incorporate the distinct behavior of clay-bound water. Future developments should aim to bridge microstructural water dynamics with macroscopic mechanical responses, enabling more realistic simulation of clay-rich unsaturated soils under thermal, hydraulic, and mechanical loading paths.

### 6.5 Models Incorporating Small-Strain Stiffness

A further refinement in the constitutive modeling of unsaturated soils is the explicit incorporation of small-strain stiffness to capture the initial elastic response of soils. This aspect is particularly important for understanding early-stage deformations, where the soil response is predominantly elastic and microstructural interactions significantly influence macroscopic behavior. Models that integrate small-strain stiffness typically modify the elastic component of the constitutive law to account for the effects of suction on stiffness. This approach enhances the model's ability to predict settlement and early deformation behavior under low-load conditions. It improves the link between the microstructural behavior observed in laboratory tests and the macro-scale predictions required in engineering practice.

Accurate characterization of small-strain stiffness in unsaturated soils is critical for geotechnical applications ranging from predicting ground vibrations to evaluating early-stage settlement. At very small strains, even minute deformations can significantly alter the pore structure, water retention properties, and, ultimately, the effective stress state. Consequently, several researchers have dedicated considerable effort to developing constitutive models that explicitly capture the small-strain response of unsaturated soils, often by incorporating bounding surface plasticity concepts and advanced stiffness scaling techniques.

One influential contribution in this field is the work by Zhou et al. (2015), who developed a bounding surface plasticity model specifically tailored for unsaturated soils at small strains. Their model emphasizes the need to accurately reproduce the anisotropic, nonlinear stiffness characteristics observed in laboratory tests. By defining a bounding surface that governs plastic deformation even at minute strain levels, the model successfully captures the rapid changes in shear modulus as the soil transitions from an initially stiff state to one that progressively softens with increasing deformation.

Similarly, Wong and Mašín (2014) developed a coupled hydro-mechanical model for partially saturated soils that predicts small-strain stiffness with particular attention to the effects of void ratio, degree of saturation, and suction. Their study demonstrated that the small-strain shear modulus is highly sensitive to the hydraulic history and the instantaneous suction level. Experimental observations revealed that even subtle changes in suction can lead to marked variations in stiffness. Thus, their formulation accounts for hysteretic behavior and the nonlinear scaling from small to finite strains, providing a seamless transition to the nonlinear behavior observed at larger deformations.

Traditional linear elastic assumptions often fail to capture the true response of unsaturated soils under cyclic or dynamic loads, where hysteresis and nonlinearity dominate. The enhanced model can consider shear modulus variation at small strains and provides improved predictions of both shear stiffness and the energy dissipation characteristics of soils, which are crucial for dynamic analyses and vibration-based applications.

Scaling relationships that bridge the gap between small and finite strains have also received significant attention. For example, a study on the scaling of shear modulus from small to finite strain conditions by Georgetti and Vilar (2013) established that a unified formulation could accurately predict the stiffness evolution as soils deform. Similarly, Dong et al. (2016) proposed a unified model for the small-strain shear modulus of variably saturated soils, highlighting that the effective stiffness is strongly influenced by saturation level. That model must incorporate both hydraulic and mechanical variables to achieve reliable predictions. The unified model for the small-strain shear modulus by Dong et al. (2016) illustrates how effective stress parameters must be adjusted as soils transition from elastic to nonlinear responses. This scaling is critical for ensuring continuity between the small strain behavior captured in laboratory tests and the more pronounced nonlinearity observed under higher deformations in the field. Additionally, the unified small-strain shear stiffness model proposed by Biglari et al. (2021) further emphasizes the importance of integrating both hydraulic and mechanical aspects. Their model calculates the stiffness as the product of a dimensionless stiffness index and individual functions of mean average skeleton stress, over-consolidation ratio, reference saturated void ratio, and degree of saturation. The introduction of a reference saturated state significantly improves predictions by incorporating a direct dependency of stiffness on saturation level, providing accurate modeling for both saturated and unsaturated fine-grained soils. This comprehensive formulation allows for more reliable extrapolation from laboratory conditions to field applications, especially in complex loading and saturation scenarios.

Additional insights into the variability of small-strain responses have emerged from studies addressing the reproducibility and sensitivity of measured stiffness. These investigations have noted that subtle changes in microstructure or testing conditions can induce considerable variability in small-strain stiffness measurements, underscoring the importance of incorporating robust, adaptable model formulations.

Recent comprehensive reviews, such as the one by Castellon and Ledesma (2022), have reinforced that modern soil constitutive models must accurately capture small-strain behavior to reliably predict static and dynamic responses. Their synthesis of current methodologies emphasizes the critical importance of integrating bounding surface concepts, hysteretic behavior, and advanced scaling laws to improve model fidelity.

Moreover, models focusing on the dynamic response of soils at small strains provide further evidence of the necessity to capture early-stage behavior. For example, the soil dynamic constitutive model for characterizing the nonlinear-hysteretic response developed by Chong (2017) shows that the early-stage response under cyclic loading can be markedly nonlinear.

This dynamic behavior has important implications for the design of structures subjected to vibrations and transient loads. Incorporating small-strain stiffness into constitutive models bridges a critical gap between purely elastic assumptions at low strains and fully elastoplastic behavior at higher strains. The evolution of models incorporating small-strain stiffness reflects an increasing recognition that the initial stiffness of unsaturated soils, governed by suction, void ratio, and microstructural variability, plays a crucial role in static and dynamic soil behavior. The integration of bounding surface plasticity models, advanced scaling laws, and dynamic constitutive frameworks provides a comprehensive basis for predicting the small-strain response, which is essential for accurately simulating and designing geotechnical systems under variable environmental conditions. Small-strain stiffness models (e.g., Zhou et al. (2015); Wong and Mašín (2014)) emphasize the transition from an initially stiff response to progressive softening. In these models, the elastic component of the constitutive law is refined to account for suction, void ratio, and microstructural variability. This enables a more accurate prediction of early-stage settlement and dynamic responses, which is critical for applications such as ground vibration analysis and early-stage deformation predictions.

When comparing these various approaches of constitutive modeling, several trends emerge. First, there is a clear progression toward models that are more physically based and less reliant on purely empirical calibration. Early models, while innovative, often suffered from parameter sensitivity and limited experimental validation. By contrast, more recent coupled and small strain formulations benefit from an extensive experimental database that allows for rigorous calibration, albeit at the expense of increased computational complexity. Second, advances in computational methods, such as more efficient numerical integration schemes and data-driven parameter estimation techniques, are gradually reducing the computational overhead associated with these sophisticated models, making them more accessible for practical engineering applications.

Finally, predictive accuracy has improved markedly as models have evolved. Modern formulations can better capture phenomena such as collapse upon wetting, cyclic hysteresis, and the anisotropic evolution of stiffness, thereby providing engineers with more reliable tools for design and analysis. Nonetheless, the increased complexity of these models calls for further research, particularly in developing standardized calibration protocols and validating model predictions under field conditions.

### 6.6 Applications and Practical Insights

Constitutive models that capture the unique features of unsaturated soil mechanics are integral to a wide array of geotechnical and geo-environmental applications. Research published in the literature (e.g., Zhang et al. (2014)) shows that in slope stability analyses, for instance, suction can provide significant apparent cohesion in otherwise marginally stable slopes. Still, infiltration events can diminish this strength rapidly and trigger failures. Foundations on expansive clays, where moisture fluctuations induce shrink-swell cycles, similarly benefit from predictions incorporating suction into volumetric and shear strength behavior. Unsaturated soil conditions also dominate in landfill covers, where the aim is to limit infiltration and potential desiccation cracking.

Despite such relevance, the widespread adoption of advanced unsaturated models in engineering practice remains limited by the difficulties of parameter calibration and the computational overhead of coupling hydraulic and mechanical processes. Another impediment is the lack of standardized protocols for testing and software implementation. Consequently, simplified or empirical approaches remain common, especially for routine analyses, even if they occasionally sacrifice accuracy when moisture conditions change rapidly.

## 7 Stress–dilatancy laws and cyclic loading in unsaturated soils

Critical state models for unsaturated soils typically extend Cam-Clay-type dilatancy frameworks by accounting for suction effects on the soil's state parameter or critical state line. For instance, the original BBM (Alonso et al., 1990) employed an associated flow rule where the dilatancy (ratio of plastic volumetric to shear strain) is tied to the position of the stress point relative to the suction-dependent yield surface. This approach captures major volumetric mechanisms like collapse on wetting but may not accurately reproduce all aspects of shear-induced dilatancy. Subsequent variants introduced refinements. Kohgo et al. (1993) developed one of the earliest theoretical models with separate yield surfaces for shear and volumetric changes under suction, implicitly recognizing that a non-associated flow rule might be needed to match observed volume change better. Wheeler and Sivakumar (1995) extended Cam-Clay concepts to unsaturated soils, introducing suction-controlled yield curves but initially keeping the flow rule associated. In these models, suction elevates the mean stress required to reach critical state, effectively reducing dilative tendencies at a given net stress. The critical state frictional constant $M$ is often assumed to remain the same as in saturation. However, the increased cohesion from suction makes the soil less contractive for the same stress ratio.

To improve realism, state-dependent dilatancy laws were later proposed. Chiu and Ng (2003) introduced an explicit state parameter for unsaturated soils, extending the concept of density-dependent dilatancy to account for current void ratio and suction. In their model, the dilation rate is not fixed by a single $M$ value; instead, it evolves with the "distance" from the unsaturated critical state line (which shifts with suction). This means a loose unsaturated soil (above the critical state line for its suction) will contract until it approaches the line, whereas a dense soil can still dilate, similar to saturated behavior but modulated by suction. Such formulations acknowledge that the critical state line in p–q–s (mean stress–deviatoric stress–suction) space may move with moisture changes. Indeed, recent studies have made the critical state condition explicitly dependent on the degree of saturation or suction. For example, Li et al. (2019) proposed an elastoplastic model in which the location of the critical state line shifts as a function of saturation, allowing more accurate prediction of dilation or contraction as suction varies. Overall, critical-state-based dilatancy laws now commonly incorporate suction either through modified hardening (yield stress increases with suction) or through an unsaturated state parameter that enters the dilatancy (flow) rule. These advances mean that as suction increases, the model will predict a smaller dilation rate (or greater contraction) for a given stress ratio, reflecting experimental evidence that drier soils tend to dilate less and may even become contractive. However, classical formulations with purely associated flow often cannot perfectly fit all observed volume-change behavior, so some modern critical state models allow slight deviations (non-associativity) to calibrate the volumetric strain response. Mechanical hysteresis under purely monotonic loading is generally not captured in these early models – unloading is assumed elastic, so stress–strain loops close upon reversal except for the permanent volumetric offsets from plastic strain. Without additional mechanisms, a simple Cam-Clay-type unsaturated model would thus predict little to no plastic strain accumulation under repeated identical loading cycles, contrary to real behavior.

### 7.2 Bounding Surface Plasticity and Cyclic Loading under Suction

To address the limitations of classical models under cyclic or repetitive loading, researchers introduced bounding surface plasticity concepts for unsaturated soils. In a bounding surface model, the yield surface is no longer a fixed envelope with a sharp elastic–plastic boundary; instead, plastic strains can develop for stress states inside a bounding surface, with the magnitude of plasticity depending on proximity to that surface. This framework inherently produces mechanical hysteresis and gradual accumulation of deformation (ratcheting) under cyclic loads. Khalili et al. (2008) leveraged this concept to capture the irreversible strains of unsaturated soils during repetitive loading–unloading cycles. Using a bounding surface that contracts or expands with plastic straining, their model allows partial yielding even for stress cycles that do not reach the previous yield limit. Consequently, each loading cycle can induce some incremental plastic strain, and upon unloading and reloading, the stress–strain path exhibits a loop (hysteresis) rather than retracing purely elastically. This approach successfully reproduces cyclic ratcheting observed in unsaturated soils, where repeated loading at constant suction causes progressive compaction or shear strain accumulation with each cycle.

In unsaturated conditions, bounding surface models often include suction as a variable that enlarges or shrinks the bounding surface. Higher suction typically increases the size of the elastic domain (i.e., the soil becomes stiffer and yields at higher stress), meaning fewer plastic strains under a given cyclic load amplitude. Upon wetting (reducing suction), the bounding surface may contract, leading to sudden volumetric collapse if the stress state is outside the reduced surface – a mechanism these models naturally handle. Wheeler et al. (2003) hinted at such effects by coupling mechanical behavior with water retention, though their model remained within an associated flow framework. Later, Zhou and Sheng (2015) formally incorporated a state-dependent bounding surface for unsaturated soils that depends on initial density and suction. Their formulation allowed the yield surface to evolve smoothly with plastic strain and changing saturation, enabling more accurate predictions for complex loading–unloading paths. Notably, including initial density (void ratio) as a state variable means the dilatancy response during cyclic loading is sensitive to how far the soil is from its current critical state (similar in spirit to state-parameter models). Under one-way cyclic loading at constant suction, such a model will show a diminishing tendency to dilate with each load cycle as the soil compacts and the state approaches the critical state line. Under load reversal (e.g., switching the shear direction), a bounding surface model can exhibit immediate plasticity because the stress point near one side of the surface suddenly finds itself near the opposite side of a translated or rotated bounding surface. This produces a realistic drop in shear resistance and a burst of contractive strain upon reversal, reflecting the destruction of the previous fabric orientation – an effect difficult to capture with an isotropic yield surface.

An example of a modern bounding-surface implementation in unsaturated soils is the work of Bruno and Gallipoli (2019). They developed a combined hydraulic–mechanical model in which the mechanical part uses a bounding surface for isotropic compression, and the hydraulic part uses a hysteretic soil-water retention formulation. Although their model was restricted to isotropic stress paths (no explicit shear yield surface was defined), it demonstrated the power of bounding surfaces to reproduce hysteretic hydro-mechanical coupling: during cyclic loading–unloading at constant suction, the soil's compression curve shows different behavior on loading vs. unloading, and some irreversible volumetric strain accumulates in each cycle. Mechanical hysteresis is manifested as a plastic swelling upon unloading that does not fully follow the elastic swelling line due to the bounding surface mobilizing partial plastic strains even at lower stresses. Such concepts can be extended to general 3D stress states. Gallipoli and Bruno (2017) introduced a bounding surface compression model with a unified normal compression line for both saturated and unsaturated soils, laying the groundwork for handling structure and irreversible deformation continuously. By combining these ideas, the Bruno and Gallipoli (2019) framework provides a template wherein unsaturated dilatancy could be handled by a similar bounding surface, ensuring gradual yielding in shear.

Overall, bounding surface plasticity has proven effective at capturing cyclic behavior under suction, including ratcheting and mechanical hysteresis that simpler models miss. Unlike classical associated models, which often predict no new plastic strain until a yield criterion is exceeded, bounding surface models predict small plastic strains for sub-yield stress excursions. This means that even minor cyclic stress changes (e.g., traffic loading on an unsaturated subgrade or seismic vibrations in an unsaturated slope) can lead to some permanent deformation, accumulating over many cycles, in agreement with experimental evidence. Importantly, these models still reduce to conventional critical state behavior in monotonic loading to failure; the difference is in how they handle partial or reversed loading. Suction enters these formulations through hardening rules (e.g., making the size or position of the bounding surface a function of suction or degree of saturation) and through coupling to water-retention hysteresis. Thus, as suction varies (e.g., during wetting–drying cycles), the model can exhibit plastic deformation not only from changes in stress but also from the cyclic variation of suction itself. Modern bounding-surface models capture how dilatancy evolves under complex environmental loading by accounting for both mechanical and hydraulic hysteresis. For example, a dense unsaturated soil that was dilating during initial shearing may become contractive upon reversal or wetting as the internal structure (fabric bonding and saturation-dependent stiffness) re-adjusts – a behavior that these advanced models can simulate by allowing the yield surface to translate/rotate and the hardening to depend on the suction path.

### 7.3 Hypoplastic Models and Non-Associated Flow Behavior

An alternative approach to modeling unsaturated soil behavior is hypoplasticity, which forgoes a yield surface and flow rule in favor of incrementally nonlinear stress–strain equations that inherently capture path-dependent behavior. Hypoplastic models are typically non-associated in effect, since dilative or contractive tendencies are governed by material-specific functions rather than tied to a yield function’s normal. Early hypoplastic formulations were mostly limited to dry or saturated soils, but recent decades have seen their extension to unsaturated conditions. Mašín and Khalili (2008) presented one of the first comprehensive hypoplastic models for unsaturated soils, using Bishop’s effective stress concept to account for partial saturation and emphasizing the stiffening effect of suction on the mechanical response. In their model, the stress–dilatancy behavior emerges from the interplay of nonlinear stress–strain relations and does not require specifying an explicit dilatancy rule. This gives significant flexibility: the model can be calibrated to match observed volumetric behavior (including collapse on wetting and dilation at failure) by adjusting a handful of parameters, without strictly obeying an associated flow law. In essence, hypoplasticity allows separate control of shear strength and dilatancy. For example, one can increase the predicted peak friction angle (shear resistance) independently of the volume change rate during shearing, which is not straightforward in traditional critical state models. This non-associated character is advantageous for capturing the often subtle volume changes in unsaturated soils, such as slight dilation at low suctions transitioning to net contraction at high suctions, which can be fitted by tuning the hypoplastic strain–stress functions.

Modern hypoplastic unsaturated models also incorporate hydro-mechanical coupling and memory of previous loading (hysteresis) through internal variables. A key development is integrating a hysteretic soil-water retention model directly into the constitutive law. For instance, in the hypoplastic model by Wong and Mašín (2014), the void ratio dependence of the retention curve is included so that changes in pore-water content influence the effective stress and stiffness continuously. They introduced a variable to track wetting/drying history, enabling smooth transitions between main drying and wetting paths via intermediate scanning curve. This allows the model to simulate hydraulic hysteresis (different suction–saturation paths for drying vs. wetting) and its effect on mechanical response – for example, a soil that has been dried will exhibit higher yield stress and dilatancy due to increased suction, and upon wetting, the model captures the reduction in dilatancy and potential collapse as the suction decreases. On the mechanical side, hypoplastic models often include an intergranular strain concept (or a similar memory surface) to account for small-strain stiffness and cyclic degradation. This feature allows simulation of hysteretic stress–strain loops under cyclic loading, even in the absence of a classical yield surface. Wong and Mašín (2014) demonstrated that considering suction history in the small-strain stiffness is crucial for predicting cyclic effects: the size of the quasi-elastic range in their model contracts or expands depending on prior wetting or drying, which in turn affects how the stiffness degrades in subsequent load cycles. By capturing this dependence, their hypoplastic model reproduces the observed reduction in shear modulus after wetting and the associated increase in damping (energy dissipation) during cyclic shear phenomena critical for unsaturated dynamic analyses.

The most recent contributions by Tafili and Mašín (2023) have further advanced hypoplastic modeling for unsaturated soils. Their generalized hydro-mechanical hypoplastic model builds on earlier frameworks (e.g., Fuentes & Triantafyllidis, 2013) and incorporates the degree of saturation and soil structure as explicit state variables. Formulated in terms of effective stress and saturation, the model can seamlessly transition between saturated and unsaturated conditions. It introduces improved representation of mechanical hysteresis: because there is no fixed yield surface, unloading and reloading inherently follow different paths, producing stress–strain loops that match observed behavior under cyclic loads. Moreover, the model can simulate how dilatancy evolves upon stress reversal by including fabric anisotropy or structure degradation parameters. For example, after initial shearing in one direction, a reversal of shear direction leads to a reduced dilatancy (often initial contraction) because the previous fabric orientation is lost – a behavior the model captures through changes in the internal variables rather than an explicit yield surface rotation. The hypoplastic formulation achieves a similar outcome to kinematic hardening in elastoplastic models, but with a smooth, continuous formulation. High suction in such models increases the effective stress, which generally makes the response more dilative (or less compressive) for a given deviatoric stress increment, consistent with experimental trends. Upon wetting or suction reduction, the model naturally produces more contractive behavior as the reduced effective stress softens the soil and promotes volume decrease. These effects emerge from the constitutive equations and their dependence on suction and void ratio, rather than from switching yield surface parameters.

Hypoplastic models provide a powerful framework to capture unsaturated soil dilatancy and cyclic behavior with a non-associated flow approach. They adhere to critical state principles at the ultimate states (ensuring convergence to a unique critical state under given suction). Still, they allow much more nuanced control of the path-dependent behavior leading there. By calibrating their nonlinear laws, hypoplastic models can replicate observed phenomena such as suction-induced stiffening, wetting-collapse, cyclic hysteresis, and evolving dilatancy without needing multiple yield surfaces or an empirical flow rule. Mechanical hysteresis is inherently included – unloading is not purely elastic – so each loading cycle leaves a footprint on the subsequent response. This means dilatancy evolves with suction changes and cyclic reversal in a natural way: as suction increases, dilatancy is curbed by higher effective stress, and when the loading direction reverses, the model's internal state ensures an initial contractive response (negative dilatancy) before a new steady dilatancy regime establishes. These capabilities make hypoplastic models well-suited to predict the complex behavior of unsaturated soils under cyclic or transient loads, complementing the critical state and bounding surface approaches in modern unsaturated soil mechanics.

**Table 5.** Complete hardening and dilatancy (chronological order).

| # | Model (citation label) | Hardening mechanism | Dilatancy / flow rule |
|---|---|---|---|
| 1 | Alonso et al. (1990) | Isotropic suction-dependent hardening: "loading–collapse" (LC) yield surface expands with increasing suction. | CSL-based (Roscoe–Burland Cam-Clay) flow rule, adjusted for suction (critical state line shifts with suction). |
| 2 | Gens and Alonso (1992) | Double-structure hardening: two nested Cam-Clay type yield surfaces (macro- and micro-structure) both enlarge with suction changes (expansive clay model). | CSL shift with swell–shrink effects: critical state line position shifts due to suction, with an empirical correction for wetting-induced swelling and drying shrinkage. |
| 3 | Wheeler and Sivakumar (1995) | Isotropic Cam-Clay type hardening (yield stress increases with net mean stress and suction via Bishop's $\chi$). No special suction hardening beyond the use of effective stress. | CSL-based associated flow rule (critical-state Cam-Clay dilatancy). |
| 4 | Alonso et al. (1999) | Double-structure hardening: macrostructural yield (like BBM) plus additional microstructural yield locus for suction-induced swelling/collapse. Both yield surfaces evolve with suction (expansion on drying, contraction on wetting). | CSL-based flow rule, with critical state framework applied at both structure levels (dilatancy calculated relative to each structure's state; microstructural volume changes shift macro-level CSL). |
| 5 | Oldecop and Alonso (2001) | Isotropic volumetric hardening for rockfill: yield is defined by an empirical compressibility limit. Irreversible compression (particle breakage) occurs once a stress threshold is exceeded (no suction term; dry material). | Empirical compressibility law governs volumetric strain (no formal critical-state shear dilatancy rule, as the model focuses on one-dimensional compaction). |
| 6 | Loret and Khalili (2002) | Thermodynamic (energy-based) formulation with isotropic hardening: yield function derived from a free-energy potential; yield size | Associated flow (plastic potential = yield function), so no separate dilatancy law |

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| | | grows with plastic volumetric strain (and depends on suction through effective stress). | specified (dilatancy implicitly follows the yield surface shape). |
| 7 | Wheeler et al. (2003) | Combined kinematic and isotropic hardening: a bounding surface in stress space translates (kinematic hardening) to capture mechanical hysteresis, and a separate “hydraulic” yield surface shifts isotropically with suction changes. | CSL-based dilatancy is evaluated relative to the current active (bounding) surface position (critical-state flow rule applied within the moving yield surface). |
| 8 | Gallipoli et al. (2003) | Isotropic suction-dependent hardening: Cam-Clay type yield surface whose size increases with suction; also, elastic stiffness moduli vary with degree of saturation. | CSL-based flow rule (standard critical-state dilatancy formulation applied). |
| 9 | Chiu and Ng (2003) | Isotropic hardening with suction influence: yield stress is modified by a state parameter (void ratio relative to critical) that evolves with suction. | State-parameter dilatancy of NorSand type: dilation/contraction rate is governed by the current state’s distance from critical state (void ratio vs CSL). |
| 10 | Tamagnini (2004) | Isotropic hardening (Extended Cam-Clay): pre-consolidation pressure evolves with plastic volumetric strain and is increased by higher suction (suction elevates the yield cap). | CSL-based flow rule (classic Cam-Clay associated flow in p–q space). |
| 11 | Sheng et al. (2004) | Isotropic suction-dependent hardening with thermal effects: yield surface expands with plastic volumetric strain; suction and temperature shift the apparent pre-consolidation stress (coupled thermo-plastic yield surface). | CSL-based thermoplastic flow rule: critical-state frictional response with modifications for thermal volume change (associated flow in extended p–q–T space). |
| 12 | Fredlund and Pham (2006) | No classical yield surface; model uses an empirical volume–mass hardening approach – volumetric strain hardening is tied to water content changes (two independent stress variables formulation). | Empirical volumetric flow rule: dilatancy is not derived from CSL but from fitting soil’s volume change behavior (focus on matching observed swell/collapse rather than a theoretical flow rule). |
| 13 | Sun et al. (2007) | Isotropic hardening affected by density: yield stress evolves with plastic strain and is higher for a denser initial state (density-dependent yield function). Suction enters via the net stress term. | CSL shift by density: the critical state line (and hence dilatancy) shifts according to current density; a state parameter linked to density modifies the standard CSL-based flow rule. |
| 14 | Li (2007) | Isotropic hardening via thermodynamic potential: yield surface defined by energy conjugates; suction acts to translate/expand the yield surface in stress space (higher suction = larger yield stress). | CSL-based flow rule (critical-state derived plastic flow; effectively Cam-Clay type dilatancy governed by the critical state line). |
| 15 | Gallipoli et al. (2008) | Isotropic suction-dependent hardening: similar to Cam-Clay with effective stress; yield surface grows with suction. The model unifies behavior at normal consolidation and critical state. | CSL-consistent flow rule: follows critical-state mechanics (dilatancy vanishes at critical state). The critical state line is defined to ensure convergence of behavior as suction varies (identical frictional parameter M for all saturations). |
| 16 | Khalili et al. (2008) | Isotropic hardening with rate effect: conventional hardening plus an additional term accounting for suction change rate (captures hysteresis under cyclic wetting–drying). | Empirical cyclic dilatancy: volumetric strain increment is adjusted based on shear strain amplitude and wetting/drying cycle history (to reproduce cyclic contractive/dilative response under |

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| | | | suction cycles). |
| 17 | Sheng et al. (2008) | Isotropic hardening using independent stress variables: yield is defined separately in an extended space of net stress and suction (no single effective stress), allowing separate plastic mechanisms for changes in stress vs suction. | CSL-based associated flow rule: similar to Cam-Clay for shear (critical-state dilatancy) under net stress, while changes in suction induce plastic volume change via a separate mechanism (yield for suction changes is associated with its potential). |
| 18 | Stropeit et al. (2008) | Anisotropic hardening: yield surface is distorted based on fabric – plastic straining causes directional hardening (different yield stress in different directions). Suction is incorporated via Bishop's stress. | CSL-based (modified for anisotropy) flow rule: flow is associated with an anisotropic plastic potential; dilatancy follows a critical-state relation that is rotated/scaled according to fabric anisotropy. |
| 19 | Zhang and Ikariya (2011) | Isotropic hardening in skeleton stress space: yield surface defined in terms of "skeleton stress" (Terzaghi effective stress scaled by degree of saturation) expands with plastic volumetric strain. | CSL-based flow rule: uses a critical-state line defined in skeleton stress space, so dilatancy follows standard Cam-Clay relations concerning the skeleton stress (associated flow in p′–q space). |
| 20 | Zhou et al, (2012a, b) | Isotropic hardening in combined stress–saturation space: yield surface evolves with plastic strain and shifts with changing degree of saturation (coupled mechanical and retention behavior). | CSL shift with saturation: the dilatancy rule is based on the critical state, but the position of the CSL (or the plastic potential) is adjusted as saturation changes (to reflect softening upon wetting and hardening upon drying). |
| 21 | Liu and Muraleetharan (2012) | Isotropic hardening (Cam-Clay style) with full hydro-mechanical coupling: yield stress increases with plastic strain and suction (suction acts like an additional hardening component via an independent scanning curve). | CSL-based associated flow: critical-state type flow rule for shear, integrated with a water-retention model so that dilatancy and water uptake are consistent (plastic volumetric strain affects saturation and vice versa). |
| 22 | Lloret-Cabot et al. (2013) | Isotropic hardening with retention coupling: yield surface size depends on plastic strain and saturation via a coupling function (mechanical and retention behaviors unified). | CSL shift via coupling: flow rule follows critical-state principles.Still it is modified by a retention state variable – the dilatancy (deviatoric vs volumetric plastic flow) adjusts as a function of the soil's wetting/drying state. |
| 23 | Ghasemzadeh and Amiri (2013) | Isotropic hardening (elastoplastic framework) under suction-controlled compression: yield stress increases with plastic volumetric strain; model formulated for isotropic loading with suction as a parameter. | CSL-based flow rule: standard associated flow on the p–q plane (Cam-Clay type) for shear behavior; volumetric plastic strains due to suction change are handled empirically (since focus is isotropic compression collapse). |
| 24 | Wong and Mašín (2014) | Hypoplasticity (no distinct yield surface): stiffness and asymptotic states depend on suction history (past drying increases stiffness). Plastic deformation accumulates according to a rate law with internal variables for suction. | Implicit hypoplastic flow rule: no explicit dilatancy equation – the model's stress–strain rate equations inherently capture dilatancy, calibrated to match critical |

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| | | | state and small-strain behavior. |
| 25 | Zhou and Sheng (2015) | Isotropic hardening (advanced HM model): yield surface evolution considers initial density (compactness) – dense vs loose soils have different hardening rates. Suction affects yield stress via a standard BBM-like formulation. | State-parameter dilatancy: the flow rule is based on a state parameter (e.g., relative density or void ratio offset from critical) so that dilation is reduced in dense (overconsolidated) states and increased in loose states, consistent with critical state concepts. |
| 26 | Zhou et al. (2015) | Bounding-surface plasticity: a moving (kinematic) yield surface defined by radial mapping; the bounding surface size (radius) scales with suction (contracting on wetting). | Empirical small-strain dilatancy: a non-associative flow rule calibrated to match observed small-strain behavior, which tends toward critical state at large strains. However, at small strains, dilatancy is adjusted empirically to fit stiffness decay. |
| 27 | Tourchi and Hamidi (2015) | Isotropic hardening with thermal coupling: yield surface (critical state line in p–q–T) expands/shifts due to temperature changes (thermal softening at higher T). Suction is included via net stress. | CSL-based thermo-dilatancy: flow rule follows critical state concepts with temperature-dependent modifications (e.g., friction angle or CSL slope M changes with temperature), capturing thermal influence on dilatancy and contraction. |
| 28 | Lloret-Cabot et al. (2017) | Isotropic hardening (unified model): a single framework governs mechanical yield and retention behavior. Depending on user choice, net stress–suction or Bishop's stress is used; the yield surface expands with plastic strain and adjusts with suction. | CSL-based flow rule: classic critical-state dilatancy relation applied in a unified manner for saturated and unsaturated states (ensuring a smooth transition); the flow rule is consistent whether using net stress or effective stress, controlled by the same CSL. |
| 29 | Chong (2017) | Kinematic hardening for cyclic behavior: employs a backbone curve that shifts with loading, following Masing rules for unloading–reloading (captures mechanical hysteresis). Suction is held constant (mechanical-only model). | Empirical cyclic dilatancy: a rule linking shear strain amplitude to incremental volume change is used to reproduce cyclic dilative/contractive behavior; not derived from critical state theory, but calibrated to match hysteresis loops. |
| 30 | Gholizadeh and Latifi (2018) | Isotropic hardening with full hydro-mechanical coupling: yield surface evolves with plastic strain; suction effects enter via a scanning curve function that adjusts yield stress during wetting/drying. | CSL-based flow rule: uses a critical-state type dilatancy formulation. The presence of suction (via scanning curves) influences the plastic volumetric strain path, but the fundamental flow rule remains Cam-Clay-like. |
| 31 | Li and Yang (2018) | Isotropic hardening factoring in OCR: yield stress evolution depends on accumulated plastic strain and the soil's overconsolidation ratio (OCR) – highly overconsolidated soils have a reduced hardening rate. Suction affects yield via BBM-style laws. | CSL shift by OCR: the critical state (dilatancy) condition is adjusted based on OCR; effectively, a state parameter accounting for OCR shifts the dilatancy behavior, causing less dilation in heavily pre-consolidated (dense) |

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| | | | states and more in loose states. |
| 32 | Bruno and Gallipoli (2019) | Dual bounding surfaces: one bounding surface governs mechanical (shear) behavior and another governs hydraulic behavior; both evolve kinematically to capture hysteresis. Yielding in shear follows a moving limit surface, and suction changes move the hydraulic surface. | CSL-based flow inside bounding surface: the plastic flow direction is determined by a Cam-Clay type rule within the current mechanical bounding surface (i.e., flow rule is formulated concerningthe shifted yield surface to ensure approach to critical state). |
| 33 | Cheng et al. (2020) | Two-surface thermo-plastic hardening: combined isotropic and kinematic hardening with a thermal yield surface. There are separate yield surfaces for thermal volumetric strain and mechanical shear that interact and shift with suction and temperature. | CSL-based flow rule with thermal modification: critical state dilatancy relations are extended to include temperature effects (e.g., thermal softening of soil reduces the dilation angle), so that the flow rule depends on the current temperature and meeting critical state at large strains. |
| 34 | Xiong (2020) | Isotropic hardening in a finite-strain framework: yield surface similar to Cam-Clay, but is formulated for large deformations; suction enters via Bishop's effective stress. Hardening law (compression index) is adapted for finite strain kinematics. | CSL-based flow rule: uses standard critical-state (associated) dilatancy, implemented for finite deformation (the form of the flow rule is unchanged, but applied in an updated Lagrangian manner for large strains). |
| 35 | Mahmoodabadi and Bryson (2021) | Isotropic hardening with hydraulic hysteresis coupling: yield surface expands with plastic strain and follows scanning curves for suction (i.e., yield stress is higher on drying paths than wetting). The fully coupled model computes changes in saturation that effect hardening. | CSL-based associated flow: a Cam-Clay style flow rule governs shear dilatancy. The coupling ensures that as suction changes (following drying/wetting scanning curves), the dilatancy prediction remains consistent with the current saturation state (contractive on wetting, etc., but still rooted in CSL). |
| 36 | Moghaddasi et al. (2021) | Bounding-surface hardening with bond degradation: incorporates soil structure (bonding) that degrades upon loading. The bounding surface (yield envelope) gradually shrinks as bonds break. Suction affects bonding and yield size (higher suction preserves bonds, expanding the surface). | CSL-based with bonding factor: the dilatancy rule follows critical-state principles. Stillit is scaled by a bonding factor (to reduce dilation in bonded (structured) soil and increase as bonding diminishes). In other words, the plastic flow is defined so that, at an equal state, bonded soil dilates less until bonds are broken, converging to the normal CSL. |
| 37 | Tafili and Machaček (2023) | Hypoplasticity with coupled hysteresis: no fixed yield surface; hardening is implicit via evolution of internal variables (fabric, solid stiffness) that capture both hydraulic and mechanical hysteresis. Suction changes modify an internal structure parameter (for collapse). | Hypoplastic flow rule: implicit non-linear flow defined by a stress–strain rate equation; dilatancy is not explicit but results from the formulation ensuring approach to a critical state. The model is calibrated so that under steady shearing, it reproduces a critical-state-like condition (zero volume |

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| | | | change). |
| 38 | Lu et al. (2023) | Isotropic chemo-thermo-hardening: extends the BExM framework to chemical and thermal effects. Yield surface shifts with suction, temperature, and chemical concentration (e.g., salinity); higher concentration or drying increases yield stress. | CSL-based with chemical scaling: chemical variables modify critical state line (e.g., reduced friction or altered volumetric strain due to chemical softening). The flow rule remains critical-state based, but parameters like M or the reference void ratio are adjusted by chemical influence. |
| 39 | Corman (2023) | Multi-scale isotropic hardening: separate consideration of macro-scale soil deformation and micro-scale gas entry. Yielding occurs when gas pressure overcomes retention capacity (pore entry pressure), and when the soil skeleton reaches its yield. The model combines these via a coupled plasticity formulation. | Empirical flow characterization: there is no single neat dilatancy rule – the model uses a pressure-dependent retention curve to dictate gas-induced volume change, and empirical relations for deformation due to gas migration. Shear dilatancy in the soil skeleton still follows basic critical-state ideas, but the dominant volumetric changes come from gas flow effects (treated empirically). |
| 40 | Sojoudi and Li (2023) | Isotropic thermo-plastic hardening: extension of Cam-Clay to include temperature-dependent yield stress. As temperature rises, yield stress (and apparent cohesion) may reduce, simulating thermal softening; suction is implicitly considered in pore pressure changes (model primarily for saturated clays with dehydration). | CSL-based thermo-plastic flow: uses an associated Cam-Clay type flow rule. At elevated temperatures, the CSL may shift (e.g., the friction angle may be slightly reduced or volumetric thermal expansion may occur). still the fundamental dilatancy relation remains tied to the critical state (adjusted for thermal volume change). |
| 41 | Quevedo et al. (2024) | Isotropic suction-dependent hardening with compaction effects: yield surface expands with plastic strain and suction, and an additional compaction variable accounts for densification (pre-compression) of soil structure due to compactive effort. | CSL-based flow rule: standard critical-state dilatancy, ensuring the model meets a unique critical state. The compaction parameter influences the plastic modulus but not the basic form of the dilatancy law, so the flow rule remains that volumetric strain drives the state toward the CSL (with faster hardening if pre-compacted). |
| 42 | Yang et al. (2024) | Hypoplastic with structural collapse factor: no explicit yield surface; an internal variable represents soil structure, which decays upon wetting (causing collapse). As suction decreases, the structure parameter drops, producing large plastic strains even without external yield violation. | Implicit hypoplastic dilatancy: dilative or contractive response emerges from the constitutive rate equations. There is no explicit dilatancy equation; however, the formulation is calibrated so that the soil approaches critical state under shearing. Wetting-induced collapse is handled by the internal structural variable rather than a conventional flow rule. |

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| 43 | Kadivar et al. (2024) | Hyperelastic bounding-surface hardening: a bounding surface plasticity model with hyperelastic (nonlinear elastic) deformation inside yield. The bounding surface size and shape depend on suction (unsaturated effective stress), translating smoothly during loading (no distinct corners). | CSL-based within bounding surface: the plastic potential is derived from a critical-state compatible surface. Essentially, dilatancy is governed by a "dilatancy surface" parallel to the CSL within the bounding surface framework, ensuring that ultimate states reach the critical state line. |

## 8 Current Trends and Emerging Topics

In recent years, an increasing focus has been on coupled thermo-hydro-mechanical-chemical (THMC) processes in unsaturated soils. This focus arises in contexts such as nuclear waste disposal, geothermal systems, and bio-mediated ground improvement, where heat, chemical reactions, or microbial activity can significantly alter soil suction regimes (Gens et al. (2010); Cheng et al. (2020); Zhang and Ikariya (2011)). Numerical modeling of THMC processes typically demands specialized laboratory experiments capable of controlling multiple variables simultaneously and significant computational resources to handle multi-field coupling.

Another prominent trend is the growing use of discrete element methods (DEM) and multi-scale frameworks. DEM simulations offer particle-scale insights, including capillary bridge formation and evolving soil fabric under variable moisture (Mitarai and Nori (2006); Li et al. (2022)). Multi-scale homogenization techniques similarly aim to connect pore-level phenomena to macroscopic constitutive laws, refining or replacing classical plasticity models (Dieudonné (2016); Corman (2023); Corman and Collin (2023)). Parallel advances in machine learning and other data-driven approaches present new avenues for parameter calibration and uncertainty quantification (Zhang et al. (2024); Zhu et al. (2022); Jia et al. (2019); Zhang et al. (2021); Zhang et al. (2021); Kirts et al. (2018)), particularly if they can be integrated with the underlying physics of unsaturated soil behavior. Zhang et al. (2021) state that ML-based models can overcome some significant limitations of conventional models, such as restrictive applicability to specific soil types and complex parameter calibration. The ML algorithms are categorized into groups such as genetic programming, evolutionary polynomial regression, support vector machines, and neural networks, including backpropagation (BPNN), radial basis function (RBF), recurrent neural networks (RNN), and their advanced variants like Long Short-Term Memory (LSTM) and Gated Recurrent Unit (GRU). Among these, LSTM and GRU are particularly noted for their suitability in capturing sequential data and stress history effects, which are crucial for accurately modeling soil behavior under cyclic loading conditions. Zhang et al. (2021) demonstrated that LSTM networks and their variants possess superior predictive accuracy and generalization abilities compared to other algorithms, making them highly effective for developing robust constitutive models. However, several challenges and limitations in current ML-based approaches remain, such as dependency on large and high-quality datasets, potential overfitting issues, and the need for further validation in complex, real-world scenarios. Zhang et al. (2021) provide constructive suggestions for improving the performance of ML-based constitutive models, including advanced training strategies, appropriate feature selection, and methods to prevent overfitting, enhancing their practical applicability and reliability in geotechnical engineering.

Growing concerns about sustainability

and climate resilience also highlight the need for robust, unsaturated models. More extreme weather patterns can induce large fluctuations in near-surface suction, increasing the risk of slope failures or infrastructure damage. Thus models that can handle transient boundary conditions and extended timeframes are critical for designing resilient geotechnical systems in a changing climate.

## 10 Research Gaps

Despite substantial progress, multiple uncertainties and open questions persist. Whether Bishop's parameter $\chi$ can fully unify unsaturated and saturated stress concepts in a single equation (Eq. (1)) remains unresolved, particularly for materials exhibiting pronounced hysteresis, microstructural complexity, or bound water effects. Many models simplify or omit hysteresis in the soil–water characteristic curve (SWRC). However, laboratory and field evidence indicates that repeated wetting–drying cycles, especially in expansive clays, alter soil behavior in ways single-valued SWRC formulations cannot capture.

An emerging research gap lies in the limited treatment of clay-bound water in constitutive models. Studies have shown that bound water, especially in smectite-rich clays, has unique structural, thermodynamic, and mechanical characteristics that are not reflected in traditional retention or strength formulations. Models that assume constant water density or treat all pore water as freely draining may significantly misrepresent suction, swelling pressure, and thermal collapse behaviors. While some recent models have begun to incorporate bound water dehydration mechanisms, their application remains narrow, and the parameterization of bound water effects is still underdeveloped.

Advanced models, such as those incorporating hierarchical yield surfaces or multi-physics coupling, often require extensive parameter sets, which can be logistically difficult and expensive to obtain. Bridging laboratory tests to field conditions poses further complications; natural soils are rarely homogeneous or static, and infiltration events or fluctuating groundwater levels may lead to transient suction and temperature profiles that do not align neatly with controlled test conditions.

Progress in modeling will require improved laboratory methods capable of distinguishing between bound, capillary, and free water phases—ideally through high-resolution imaging or spectroscopic techniques under suction and thermal control. Additionally, the integration of microstructural water behavior—such as water layering, variable density, and dehydration kinetics—into macroscale constitutive frameworks remains an open and important frontier.

Despite machine learning (ML) advancements in constitutive modeling, significant gaps remain in its application to unsaturated soils. Current ML-based models predominantly focus on saturated conditions or simple stress paths, neglecting complexities introduced by suction, hysteresis, and clay-bound water behavior. The scarcity of comprehensive datasets that account for microstructural water effects and the lack of integrated ML-hydraulic frameworks further underscore the urgent need for focused research that blends data-driven and physics-based approaches.

## 11 Conclusion

This comprehensive review of constitutive modeling for unsaturated soils underscores significant strides in capturing the complex interactions among solid, water, and air phases within soils. Over recent decades, theoretical, experimental, and computational advancements have transitioned from empirical modifications of saturated models to sophisticated frameworks incorporating elasto-plasticity, multi-scale approaches, and data-driven methodologies. Despite remarkable progress, challenges remain, particularly in selecting appropriate stress state variables, integrating coupled hydro-mechanical processes, and accurately modeling hysteresis and anisotropy.

The historical evolution highlighted in this paper illustrates key milestones, such as the introduction of dual-stress state variables by Fredlund and Morgenstern and the influential Barcelona Basic Model (BBM) by Alonso et al. These models have formed the foundation upon which contemporary constitutive frameworks have been developed, addressing more intricate phenomena like wetting-induced collapse, cyclic loading, and anisotropic behavior. Furthermore, recent trends have emphasized the importance of integrating thermo-hydro-mechanical-chemical (THMC) interactions, discrete element modeling, and advanced computational techniques like machine learning, which hold the potential for revolutionizing parameter calibration and predictive accuracy.

Machine learning, in particular, emerges as a transformative tool capable of modeling complex soil behaviors directly from data without pre-imposed theoretical constraints. However, its application to unsaturated soils remains limited due to challenges in capturing the nuanced effects of suction, hydraulic hysteresis, and coupled mechanical-hydraulic interactions. The scarcity of extensive, high-quality datasets specific to unsaturated conditions represents a critical gap, highlighting an urgent need for targeted research and validation efforts.

The review also identifies significant practical barriers to the widespread adoption of advanced constitutive models, including complex parameter determination, computational intensity, and lack of standardized testing protocols. These obstacles underline the necessity of interdisciplinary collaborations and systematic long-term field validations to enhance the reliability and applicability of these models in real-world scenarios.

Future research should prioritize addressing these identified gaps by integrating cutting-edge technologies, comprehensive data collection initiatives, and interdisciplinary cooperation. Progress in this domain is academically enriching and crucial for the sustainable design and resilience of geotechnical and geo-environmental systems under changing environmental conditions. Ultimately, continued advancements in constitutive modeling of unsaturated soils promise to significantly improve our capacity to predict, manage, and mitigate soil-related challenges across various engineering applications.

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